



INTERNATIONAL A-LEVEL MATHEMATICS

MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

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Version 1.0 Final



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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	A random variable	E1	
	that is a function of known observations from a population	E1	oe
		2	
1(b)	Range of values [of the test statistic]	E1	oe
	that leads us to determine whether or not the null hypothesis is to be rejected or not	E1	oe
		2	
	Total	4	

Q	Answer	Marks	Comments
2(a)	$\lambda = 0.1$	B1	
	$P(T < 5) = 1 - e^{-0.1 \times 5}$	M1	ft their value for λ
	$= 0.3935$	A1	AWRT 0.3935
		3	
2(b)	$P(8 < T < 14)$		
	$= (1 - e^{-0.1 \times 14}) - (1 - e^{-0.1 \times 8})$	M1	Difference between two probabilities with at least one correct probability
	$[= 0.75340... - 0.55067...]$		
	$= 0.2027$	A1	AWRT 0.2027
		2	
	Total	5	

Q	Answer	Marks	Comments
3(a)	$\int_0^a \frac{4}{(2x+1)^2} dx = 1$ $\left[\frac{4}{2 \times -1} (2x+1)^{-1} \right]_0^a = 1$ $\frac{-2}{2a+1} - (-2) = 1$ $a = 0.5$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Correct definite integral set equal to 1 Condone missing dx</p> <p>Correct integration oe</p> <p>ft from their integration with correct limits substituted</p> <p>AG Be convinced</p>
		4	
3(b)	$P(0.2 < X < 0.4) = \int_{0.2}^{0.4} \frac{4}{(2x+1)^2} dx$ $= \left[\frac{4}{2 \times -1} (2x+1)^{-1} \right]_{0.2}^{0.4}$ $= (-2) \times (2 \times 0.4 + 1)^{-1} - (-2) \times (2 \times 0.2 + 1)^{-1}$ $= \frac{20}{63}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Correct definite integral Condone missing dx PI</p> <p>Applies limits to their integration from part (a) PI</p> <p>oe, AWRT 0.317</p>
		3	
	Total	7	

Q	Answer	Marks	Comments
4	$H_0: \mu = 16$ $H_1: \mu < 16$ $\bar{x} = 15.5$ $s^2 = \frac{1}{200-1} \left(50300 - \frac{3100^2}{200} \right)$ $= 11.3[0653266]$ $\bar{X} \sim N \left(16, \frac{11.3}{200} \right)$ $z = \frac{15.5-16}{\sqrt{\frac{11.3}{200}}}$ $= -2.1[0351581]$ $z_{\text{critical}} = -2.3263$ <p>As $z > z_{\text{critical}}$ we fail to reject H_0</p> <p>Evidence to suggest that the laptop battery time between charges has not decreased</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1</p>	<p>Both hypotheses</p> <p>PI by correct calculation for z</p> <p>Attempt at variance formula Allow one slip PI by correct answer</p> <p>AWRT $\left(\frac{2250}{199} \right)$ Accept $s = 3.36[2518297]$</p> <p>$\bar{X} \sim N \left(16, \frac{s^2}{200} \right)$ PI</p> <p>Calculates z with their s^2</p> <p>AWRT -2.1</p> <p>AWRT -2.3 or $P(z < -2.1) = 0.0177$ Accept 2.3</p> <p>Follow through their z and z_{critical} provided signs are consistent or comparing 0.0177 to 1%</p> <p>Must be consistent with their conclusion on whether to fail to reject H_0 or not or their z and z_{critical} if not explicitly stated</p>
	Total	10	

Q	Answer	Marks	Comments
5(a)	Symmetrical Mode = Mean = Median 95% of the data lies within 2 sd of the mean	B1 B1 B1	oe such as 'bell-shaped curve' or 'no skew' Accept » instead of = Accept similar accurate comment eg 99% \supset 3 sd 68% \supset 1 sd
		3	
5(b)(i)	$P(X > 17) = P\left(Z > \frac{17-14}{1.2}\right)$ $= P(Z < -2.5)$ $= 1 - 0.99379[03346]$ $= 0.0062$	M1 M1 A1	PI Standardises PI by sight or use of 0.99379... AWR 0.0062
		3	
5(b)(ii)	$\text{Var}(\bar{X}) = \frac{1.2^2}{50} \quad \text{or} \quad \text{sd} = \frac{1.2}{\sqrt{50}}$ $P(\bar{X} < 13.8) = P\left(Z < \frac{13.8-14}{\sqrt{0.0288}}\right)$ $= P(Z < -1.1785)$ $= 1 - 0.8807[035853]$ $= 0.119$	B1 M1 m1 A1	PI Accept $\text{Var}(\bar{X}) = 0.0288$ or AWR sd = 0.17 PI Standardises using their $\text{Var}(\bar{X})$ but not 1.2^2 PI by correct answer AWR 0.119
		4	
5(b)(iii)	$X + Y \sim N(44, 1.2^2 + 4^2)$ $P(X + Y < 35) = P\left(Z < \frac{35-44}{\sqrt{17.44}}\right)$ $= P(Z < -2.1551)$ $= 1 - 0.9844[233527]$ $= 0.0156$	B1 M1 m1 A1	PI PI Standardises with their sum of means and their sum of variances PI by correct answer AFW 0.01539 to 0.01578
		4	
	Total	14	

Q	Answer	Marks	Comments
6(a)	$P(X = 3) = \frac{e^{-4} \times 4^3}{3!}$ $= 0.195$	M1 A1	PI May use tables: 0.4335 – 0.2381 AWRT 0.195
		2	
6(b)	$\lambda = 2 \times (4 + 2.5)$ $\lambda = 13$ $H_0: \lambda = 13$ $H_1: \lambda < 13$ $P(X \leq 7)$ $P(X \leq 7) = 0.054$ $0.054 > 0.05$ <p>Do not reject H_0</p> <p>Evidence to suggest that there has not been a reduction in the total number of breakdowns of boats and buses</p>	M1 A1 B1 M1 A1 M1 A1ft E1	PI Both hypotheses Allow $H_0: \lambda = 6.5$ $H_1: \lambda < 6.5$ Attempts $P(X \leq 7)$ or $P(X < 7)$ AWRT 0.054 Compares their probability with 0.05 ft their probability compared with 0.05 Implied by correct conclusion in context Must be consistent with their conclusion on whether or not to reject H_0 or on their probability if not explicitly stated
		8	
	Total	10	

Q	Answer	Marks	Comments
7(a)	$X \sim B(100, 0.03)$ $100 \times 0.03 = 3$ $Y \sim Po(3)$ $P(Y \leq 3)$ $= 0.647$	B1 B1 M1 A1	PI by use of $\lambda = 3$ with Poisson distribution Identifies correct approximate distribution Identifies correct probability AWRT 0.647
		4	
7(b)(i)	$H_0: p = 0.03$ $H_1: p \neq 0.03$ $X \sim B(20, 0.03)$ $P(X \geq 3) = 0.021$ $0.021 < 0.025$ Reject H_0 Evidence to suggest that the proportion of viewers watching the local news programme has changed	B1 M1 A1 M1 A1ft E1	Both hypotheses PI by a binomial probability calculation AWRT 0.021 Compares their probability with 0.025 Follow through their probability Implied by correct conclusion in context Must be in context, must not be definite and all the previous 5 marks must have been awarded.
		6	
7(b)(ii)	A Type I error means to reject that the proportion of viewers watching the local news programme is 3% when it is 3%	E2	oe E1 for describing Type I error without context
		2	
7(b)(iii)	$P(X \geq 3) = 0.021$ [< 0.025] $P(X \geq 2) = 0.1198$ [> 0.025] $P(X \leq x) < 0.025$ $P(X = 0) = 0.5438$ [> 0.025] $P(\text{Type I error}) = 0.021$	M1 M1 A1	Considers both probabilities PI by calculation $P(X = 0)$ or $P(X \leq 0)$ oe
		3	
	Total	15	

Q	Answer	Marks	Comments
8(a)	$\frac{0.4}{2}x \text{ or } 0.2x$ $y - 0.4 = \frac{0.6}{4}(x - 2)$ or $y - 1 = \frac{0.6}{4}(x - 6)$ $0.15x + 0.1$ $F(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \leq x < 2 \\ 0.15x + 0.1 & 2 \leq x < 6 \\ 1 & x \geq 6 \end{cases}$	B1 M1 A1 B1 A1	Seen anywhere Use of straight line methods to find second line Allow $y = mx + c$ methods only if a value is found for c Seen anywhere 0 when $x < 0$ and 1 when $x \geq 6$ Allow either strict or non-strict inequalities Completely defined function in ACF Allow different but consistent placement of strict inequalities
		5	
8(b)	$f(x) = \begin{cases} 0.2 & 0 \leq x < 2 \\ 0.15 & 2 \leq x < 6 \\ 0 & \text{otherwise} \end{cases}$	B1 M1 A1ft	0 and otherwise oe seen 0.2 oe or 0.15 oe seen anywhere ft their equations of lines from part (a) Completely defined function in ACF ft their equations of lines from part (a)
		3	

Q	Answer	Marks	Comments
8(c)	$E(X^3) = \int_0^2 0.2x^3 dx + \int_2^6 0.15x^3 dx$ $= \left[\frac{0.2x^4}{4} \right]_0^2 + \left[\frac{0.15x^4}{4} \right]_2^6$ $E(X^6) = \int_0^2 0.2x^6 dx + \int_2^6 0.15x^6 dx$ $= \left[\frac{0.2x^7}{7} \right]_0^2 + \left[\frac{0.15x^7}{7} \right]_2^6$ $E(X^3) = 48.8 \text{ or } E(X^6) = \frac{209984}{35}$ $\text{Var}(X^3) = E(X^6) - E(X^3)^2$ $\text{Var}(X^3) = \frac{209984}{35} - 48.8^2$ $\text{Var}(X^3) = 3620$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>Identifies correct integral for their $f(x)$</p> <p>Correct integration</p> <p>Identifies correct integral for their $f(x)$</p> <p>Correct integration</p> <p>PI, AWRT 6000 for $E(X^6)$</p> <p>Uses variance formula with their expectation values for $E(X^6)$ and $E(X^3)$</p> <p>AWRT 3620</p>
		7	
	Total	15	