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MATHEMATICS

MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	Exponential	**B1**	
		1	

Q	Answer	Marks	Comments
1(b)	$\left[\dfrac{1}{1/7}=\right]\ 7$	**B1**	
		1	

Q	Answer	Marks	Comments
1(c)	$\left[\mathrm{P}(X<7)-\mathrm{P}(X<2)\right]$ $=\left(1-\mathrm{e}^{-\frac{1}{7}\times 7}\right)-\left(1-\mathrm{e}^{-\frac{1}{7}\times 2}\right)$	**M1**	**PI** Attempts to find correct probability using cdf of exponential or integration of pdf
	$=0.3836$	**A1**	**AWRT 0.384**
		2	

Q	Answer	Marks	Comments
1(d)	$\left[\mathrm{P}\left(X < a\right) = 0.8\right] = \left(1 - \mathrm{e}^{-\frac{1}{7}a}\right) = 0.8$	M1	oe, PI
	$= 11.27$	A1	AWRT 11.3
		2	

Q	Answer	Marks	Comments
1(e)	$\left[\mathrm{P}\left(X > 8 \middle\| X > 5\right) = \mathrm{P}\left(X > 3\right) = \right] \ \mathrm{e}^{-\frac{1}{7}\times 3}$	M1	oe, PI
	$= 0.6514$	A1	CAO
		2	

	Question 1 Total	**8**	

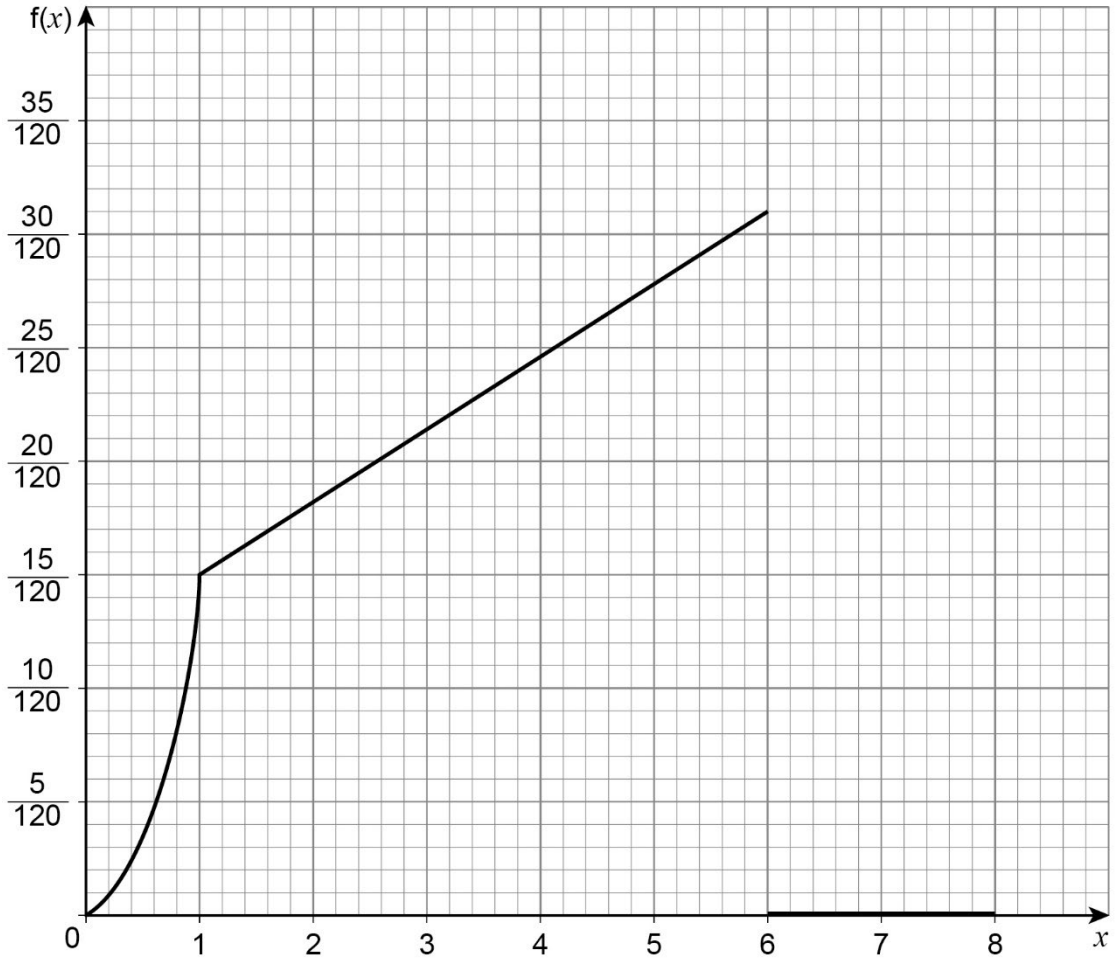
Q	Answer	Marks	Comments
2(a)	$H_0 : \mu = 300$ $H_1 : \mu > 300$	**B1**	
	$\overline{X} \sim N\left(300, \frac{40^2}{200}\right)$	**B1**	**PI** by correct standardisation formulae
	$z = \frac{306-300}{\frac{40}{\sqrt{200}}}$	**M1**	$z = \frac{306-300}{\text{their } \frac{\sigma}{\sqrt{n}}}$ **PI** by correct z or probability
	$z = 2.1213...$	**A1**	**AWRT** 2.12 or exact value $\frac{3\sqrt{2}}{2}$ **or** $P(z > 2.1213...)$
	$z_{\text{critical}} = 2.0537$	**B1**	**AWFW** 0.0169... to 0.0170 **or** comparison of probability $0.0169...$ to $0.0170 < 0.02$ Allow 'Accept H_1'
	Reject H_0 as $z_{\text{critical}} < z$ **or** $2.0537 < 2.1213\ldots$ **or** $\lvert z \rvert > 2.0537$	**A1ft**	Comment about H_0 and comparison $0.0169...$ to $0.0170 < 0.02$ Correct conclusion based upon ft their z
	There is sufficient evidence to support the claim that the average time for which the app is used has increased since the new version was released [at the 2% level of significance]	**E1**	Correct statement must be in context and must follow from fully correct solution Condone definite statement
		7	

Q	Answer	Marks	Comments
2(b)	Central limit theorem (CLT) states that [when the sample size is large enough], the sample mean will be approximately normally distributed	B1	
	A sample of 200 is large enough for the CLT to apply	B1	Allow $n \geq 30$
		2	

	Question 2 Total	**9**	

Q	Answer	Marks	Comments
3(a)	$\left[\int_0^1 \frac{x^2}{8}\mathrm{d}x\right] = \left[\frac{x^3}{24}\right]_0^1$	M1	**PI** Correct integration and limits
	$= \frac{1}{24}$	A1	**oe** in exact form
		2	

Q	Answer	Marks	Comments
3(b)	$\frac{1}{24} + \int_1^6 \left(k(x-1) + \frac{1}{8}\right)\mathrm{d}x = 1$	B1	Setting an integral and fraction summing to 1 **oe PI**
	$\frac{1}{24} + \left[\frac{k(x-1)^2}{2} + \frac{1}{8}x\right]_1^6 = 1$ $\frac{1}{24} + \left(\frac{25k}{2} + \frac{3}{4}\right) - \left(0 + \frac{1}{8}\right) = 1$	M1	For correct integration with attempt to substitute limits **oe**
	$\frac{25k}{2} = \frac{1}{3}$		
	$k = \frac{2}{75}$	A1	**AG** must be convincingly shown
		3	

Q	Answer	Marks	Comments
3(c)		B1	Curve from (0,0) to (1, 15/120) can be identified as coordinates or from their scale
		B1	Straight line from (1,15/120) to (6, 31/120) can be identified as coordinates or from their scale
		B1	Straight line from (6,0) to (8,0)
			
		3	

Q	Answer	Marks	Comments
3(d)	$\frac{x^3}{24}$	B1	
	$\int_1^x \left(\frac{2(x-1)}{75}+\frac{1}{8}\right)\mathrm{d}x+\frac{1}{24}$	M1	Correct integration condone omission of $\frac{1}{24}$
	$=\left[\frac{(x-1)^2}{75}+\frac{1}{8}x\right]_1^x+\frac{1}{24}$		
	$=\frac{(x-1)^2}{75}+\frac{1}{8}x-\frac{1}{12}$	A1	$\mathrm{F}(x)=\frac{1}{75}x^2+\frac{59}{600}x-\frac{7}{100}$ from $f(x)=\frac{2}{75}x+\frac{59}{600}$
	$\mathrm{F}(x)=\begin{cases}0 & x<0\\ \frac{x^3}{24} & 0\le x<1\\ \frac{(x-1)^2}{75}+\frac{1}{8}x-\frac{1}{12} & 1\le x\le 6\\ 1 & x>6\end{cases}$	A1	**oe** Fully correct $\mathrm{F}(x)$
		4	

	Question 3 total	**12**	

Q	Answer	Marks	Comments
4(a)	$\mathrm{B}(100,\ 0.05)$	**B1**	Condone F
	It is the only one with **large** n and **small** p	**B1**	If **B0 B0**, then allow **SC1** for comparing means and variances
		2	

Q	Answer	Marks	Comments
4(b)(i)	$\mathrm{P}(X=3)=\dfrac{\mathrm{e}^{-1.8}\times 1.8^3}{3!}$	**M1**	**PI**
	$=0.161$	**A1**	**AWRT** 0.161
		2	

Q	Answer	Marks	Comments
4(b)(ii)	$\lambda\left[=\dfrac{1.8}{3}\right]=0.6$	**B1**	**PI**
	$\mathrm{P}(1<X<4)=\mathrm{P}(X\leq 3)-\mathrm{P}(X\leq 1)$	**M1**	**PI** At least one correct probability **or** finds $P(X=2)$ **and** $P(X=3)$
	$=0.9966-0.8781$	**M1**	Correct method
	$=0.119$	**A1**	**AWRT** 0.119
		4	

Q	Answer	Marks	Comments
4(c)(i)	$\lambda\ [=1.8+2.7]=4.5$	B1	
		1	

Q	Answer	Marks	Comments
4(c)(ii)	$P(G<8)=0.9134<0.95$	M1	Sight of at least relevant probability statement
	$P(G<9)=0.9597>0.95$	A1	Sight of both probabilities and comparison to 0.95
	Hence $a=9$	B1	
		3	

	Question 4 Total	12	

Q	Answer	Marks	Comments
5(a)	$H_0 : \mu = 13$ $H_1 : \mu \neq 13$	**B1**	Both hypotheses
	$\bar{x} = \frac{129.5}{10} = 12.95$	**B1**	
	$s^2 = \frac{1}{10-1}\left(1677.05 - \frac{129.5^2}{10}\right)$	**M1**	Attempt at variance formula Condone one error **PI** by correct answer
	$= \frac{1}{360} = 0.002\dot{7}$	**A1**	**AWRT** 0.00278 Accept s = 0.0527[0462767]
	$\bar{X} \sim N\left(13, \frac{0.002\dot{7}}{10}\right)$	**M1**	$\bar{X} \sim N\left(13, \frac{s^2}{10}\right)$ **PI**
	$t = \frac{12.95 - 13}{\sqrt{\frac{0.002\dot{7}}{10}}}$	**M1**	Calculates z with their s^2
	$= -3$	**A1**	
	$v = 9 \Rightarrow t_{\text{critical}} = \pm 3.250$	**B1**	or sight of p=0.00747 **oe**
	Do not reject H_0 as $\lvert t \rvert < 3.250$	**A1ft**	Follow through their t and t_{crit} provided signs are consistent Implied by correct conclusion in context (or comparison of p to 0.005)
	Insufficient evidence to suggest that Alice's mean maximum exposure time has changed at the 1% level of significance	**E1**	Must be in context, must not be definite and all the previous 9 marks must have been awarded
		10	

Q	Answer	Marks	Comments
5(b)	He uses a different significance level or He uses a one-tailed test	**E1**	A correct suggestion
		1	

	Question 5 Total	**11**	

Q	Answer	Marks	Comments
6(a)	$\overline{V} \sim \text{N}\left(502, \frac{2.7^2}{30}\right)$	B1	PI
	$\text{P}\left(\overline{V} < 501\right) = \text{P}\left(z < \frac{501-502}{\frac{2.7}{\sqrt{30}}}\right)$	M1	PI
	$\text{P}\left(z < -2.0286...\right)$		
	$= 1 - 0.97882$ [from tables]	M1	Calculator method gives $1 - 0.97875$ PI $1 - $ 'p' where p is from a standardised value using $n = 30$
	$= 0.0212$ (to 4 dp)	A1	AWFW 0.02118 to 0.0213 SC1 AWFW 0.355 to 0.356
		4	

Q	Answer	Marks	Comments
6(b)	$\text{P}(V > 496) = \text{P}\left(z > \frac{496-502}{2.7}\right)$	M1	PI
	$= \text{P}\left(z > -2.22...\right)$		
	$= 0.98679$ [from tables]	A1	AWRT 0.987 PI
	$= \text{P}\left(\text{all } 30\right) = 0.98679^{30}$	m1	p^{30}
	$= 0.671$	A1	AWFW 0.671 to 0.673
		4	

Q	Answer	Marks	Comments
6(c)	Let $M = V + W + W + Y$ $M \sim \text{N}\left(502 + 251 + 251 + 503,\ 2.7^2 + 1.5^2 + 1.5^2 + 2^2\right)$	M1	Correct method for finding the mean or variance of M **PI** by sight of 1507 or 15.79…
	$M \sim \text{N}\left(1507,\ 15.79\right)$	A1	**PI** Fully correct distribution for M
	$\text{P}\left(M > a\right) = 0.95$ $z = -1.6449$	B1	Allow $\pm$ z value to at least 4sf
	$\frac{a - 1507}{\sqrt{15.79}} = -1.6449$	M1	**PI** Standardising with compatible signs and use of their σ
	$a = 1500$ [grams]	A1	**CAO**
		5	

	Question 6 Total	13	

Q	Answer	Marks	Comments
7(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{F}(x)=\frac{4}{x^3}$ or $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{F}(x)=\frac{x}{48}+\frac{1}{32}$	**B1**	
	$\mathrm{E}(X)=\int x\mathrm{f}(x)\mathrm{d}x$		
	$=\int_2^4 \frac{4}{x^2}\mathrm{d}x+\int_4^8\left(\frac{x^2}{48}+\frac{x}{32}\right)\mathrm{d}x$	**M1**	Correct integration with correct limits of their $x\mathrm{f}(x)$
	$=\left[\frac{-4}{x}\right]_2^4+\left[\frac{x^3}{144}+\frac{x^2}{64}\right]_4^8$		
	$=1+\frac{139}{36}$	**A1**	
	$=\frac{175}{36}$		
	$\mathrm{E}\left(X^2\right)=\int x^2\mathrm{f}(x)\,\mathrm{d}x$		
	$=\int_2^4 \frac{4}{x}\mathrm{d}x+\int_4^8\left(\frac{x^3}{48}+\frac{x^2}{32}\right)\mathrm{d}x$	**M1**	correct integration with correct limits of their $x^2\mathrm{f}(x)$ **oe**
	$=\left[4\ln(x)\right]_2^4+\left[\frac{x^4}{192}+\frac{x^3}{96}\right]_4^8$		
	$=4\ln(2)+\frac{74}{3}$	**A1**	
	$\mathrm{E}\left(X^2\right)-\mathrm{E}(X)^2=4\ln(2)+\frac{74}{3}-\left(\frac{175}{36}\right)^2$	**M1**	correct use of their $\mathrm{E}\left(X^2\right)$ and $\mathrm{E}(X)^2$
	$=4\ln(2)+\frac{1343}{1296}$	**A1**	**AG** Must be convincingly shown
		7	

Q	Answer	Marks	Comments
7(b)	$4\text{Var}(X)+9\text{Var}(Y)$ $=4\times3.81+9\times2.5^2$ $=71.5$	**M1** **A1**	 **AWRT**
		2	

	Question 7 Total	**9**	

Q	Answer	Marks	Comments
8	$H_0 : p = 0.5$ $H_1 : p \neq 0.5$	B1	Hypotheses that model the situation
	$X \sim B(20,\ 0.5)$	B1	**PI** by correct probability or $X \sim B(20, p)$
	$P(X \geq 15) = 1 - P(X \leq 14)$	M1	
	$= 0.0207$	A1	**AWFW** 0.0206 to 0.0207 **or** a CR= {0,1,2,3,4, 16,17,…,20}
	$0.0207 > 0.015$ Do not reject H_0	B1ft	Compares their probability with 0.015 (allow comparison to 0.03 if their alternative hypothesis is 1 tailed) Or compares 15 to a correct CR
	There is sufficient evidence at the 3% level to suggest that the student has randomly selected their answers	E1ft	Cannot be definitive Must follow a full hypothesis test **ft** their conclusion consistent with their model
		6	

	Question 8 Total	**6**	