OXFORDAQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS
(9660/MA04) Unit S2 Statistics

Friday 17 January 2025   07:00 GMT   Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Answer **all** questions in the spaces provided.

1 (a) Explain what is meant by a critical region for a hypothesis test.

[1 mark]

1 (b) (i) The random variable X is known to have a binomial distribution such that

$$X \sim B(50, p)$$

A hypothesis test is carried out at the 10% level of significance with the following hypotheses.

$$H_0 : p = 0.2$$
$$H_1 : p \neq 0.2$$

Find the critical region for the test.

Use probabilities to justify your answer.

[4 marks]

Answer _____

Do not write outside the box

1 (b) (ii) The value obtained for the test statistic is 17

State with a reason the conclusion of the test.

[1 mark]

Conclusion _____

Reason _____

_____

1 (c) (i) State the conditions under which a Poisson distribution would be a suitable approximation for a binomial distribution.

[1 mark]

_____

_____

1 (c) (ii) The random variable Y has a binomial distribution and $Y \sim B(80, 0.05)$

Show that a suitable Poisson approximation to Y is $V \sim Po(4)$

[1 mark]

_____

_____

_____

Question 1 continues on the next page

Turn over ▶

1 (d) (i) The random variable W is known to have a Poisson distribution with mean λ

A hypothesis test is carried out at the 5% level of significance with the following hypotheses.

$$H_0 : \lambda = 4$$
$$H_1 : \lambda < 4$$

Find the critical region for the test.

Use probabilities to justify your answer.

[2 marks]

Answer

1 (d) (ii) The value obtained for the test statistic is 1

State with a reason the conclusion of the test.

[1 mark]

Conclusion

Reason

11

Turn over for the next question

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Turn over ▶

2 The continuous random variable T has probability density function $f(t)$ defined by

$$f(t) = \begin{cases} ct^3 & 2 \le t \le 4 \\ \dfrac{1}{10} & 4 < t \le 6 \\ 0 & \text{otherwise} \end{cases}$$

2 (a) Show that $c = \dfrac{1}{75}$

[3 marks]

2 (b) (i) Find the exact value of $E(T)$

[3 marks]

Answer

2 (b) (ii) Hence find the exact value of $E(4-5T)$

[2 marks]

Answer _____

2 (c) (i) Use your answer to **part (b)(i)** to show that $\text{Var}(T) = 0.738$ correct to three significant figures.

[5 marks]

Question 2 continues on the next page

Turn over ►

2 (c) (ii) Find the value of $\text{Var}(4-5T)$

Give your answer to three significant figures.

[2 marks]

Answer _____

15

Turn over for the next question

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Turn over ▶

3 Nok has raced a bike at a race track a large number of times.

Based on his records, Nok's time for one lap of the race track X is normally distributed with a mean of 205 seconds and a standard deviation of 30 seconds.

Following a change in the type of tyres he uses for his bike, Nok claims his mean time for one lap of the race track has increased.

Nok records 10 randomly selected times for one lap of the race track and summarises his total time, in seconds, below.

$$\sum x = 2205$$

Nok assumes that the standard deviation is unchanged.

Test Nok's claim at the 5% level of significance.

[7 marks]

Turn over ▶

4 (a) For a Poisson distribution $X \sim \text{Po}(\lambda)$ where $\lambda > 0$ it is given that

$$P(X = 8) = \frac{125}{2688} \times P(X = 5)$$

Find the value of λ

[3 marks]

Answer _____

4 (b) The number of advertisements which appear on an app in a 30-minute period is modelled as a Poisson distribution with a mean of 10

4 (b) (i) Find the probability of less than 5 advertisements appearing in a 30-minute period of using the app.

Give your answer to three significant figures.
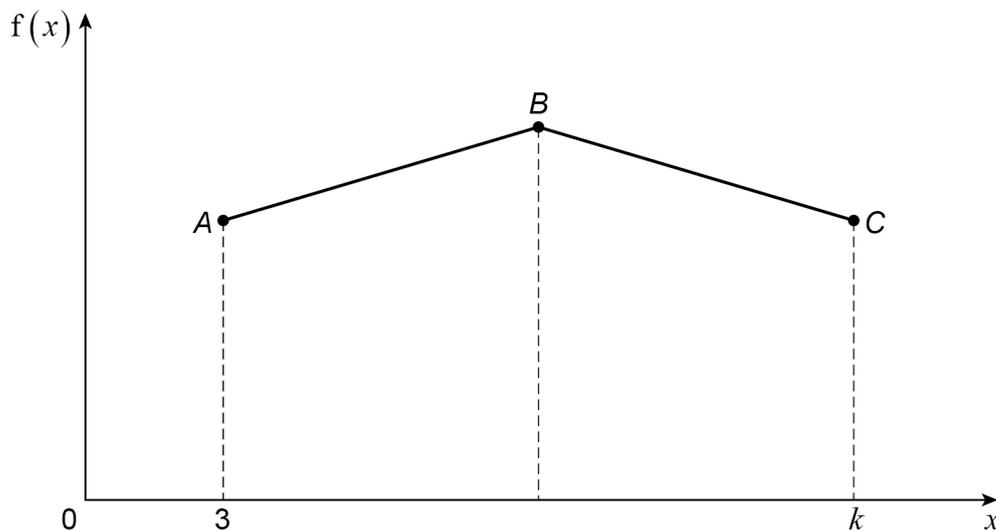
[2 marks]

Answer _____

4 (b) (ii) Find the probability of more than 7 and less than 13 advertisements appearing in a **15-minute period** of using the app.

Give your answer to three significant figures.

[3 marks]

Answer _____

4 (c) An advertisement appears at 10.15 am on the app.

Use an exponential distribution to find the probability that no more advertisements appear until after 10.20 am.

Give your answer to three significant figures.

[3 marks]

Answer _____

Turn over ▶

11

5 (a) A farmer grows pumpkins to be sold at a market.

The mass in kilograms of each of the pumpkins sold is modelled by

$$X_p \sim \text{N}\left(\mu, \sigma^2\right)$$

where μ and σ^2 are unknown constants.

Large pumpkins are defined as having a mass greater than 3.8 kg and account for 20% of the pumpkins sold.

Small pumpkins are defined as having a mass less than 1.9 kg and account for 10% of the pumpkins sold.

Find the value of μ and the value of σ

Give your values to three significant figures.

[6 marks]

$\mu =$ _____ $\sigma =$ _____

5 (b) The farmer also grows cabbages to be sold at a market.

The mass in kilograms of each of the cabbages sold is modelled by

$$X_c \sim N\left(0.80, 0.25^2\right)$$

A sample of size n is taken from the cabbages such that $P\left(\overline{X}_c < 0.82\right) > 0.74$

Find the smallest possible value of n

[4 marks]

Answer _____

10

Turn over ▶

6 (a) A continuous random variable X has probability density function $f(x)$

The graph of $f(x)$ has a vertical line of symmetry and consists of two line segments AB and BC for $3 \leq x \leq k$ where k is a constant.

For all other values of x, $f(x) = 0$ as shown in **Figure 1**.

Figure 1

6 (a) (i) It is given that $P(X < 8) = 0.5$
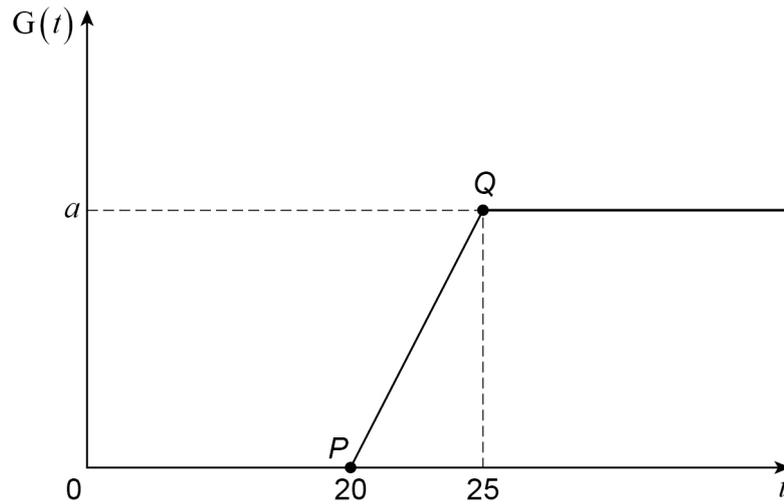
Find the value of k

[1 mark]

Answer _____

6 (a) (ii) It is also given that $f(8) = 0.125$

Find $f(x)$

[5 marks]

$$f(x) = \begin{cases} \text{\rule{5cm}{0.4pt}} \\ \text{\rule{5cm}{0.4pt}} \\ \text{\rule{5cm}{0.4pt}} \end{cases}$$

Question 6 continues on the next page

Turn over ▶

6 (b) (ii) The probability density function of T is $g(t)$

Find $g(t)$

[3 marks]

$$g(t) = \begin{cases} \text{--} \\ \text{--} \end{cases}$$

Turn over for the next question

10

7 The Zingy Sauce company supply packets of sauce to a fast food restaurant.

The mass of the packets of sauce are normally distributed.

The fast food restaurant takes a random sample of 8 packets of sauce.

For this sample, the mass in grams of the packets of sauce were recorded as

8.99   8.96   8.99   9.02   8.95   8.98   8.97   9.01

7 (a) (i) Calculate the exact value of the sample mean.

[1 mark]

Answer

7 (a) (ii) Calculate the unbiased estimate for the variance.

[2 marks]

Answer

7 (b) The Zingy Sauce company advertise that the packets of sauce have a mass of 9 grams.

The fast food restaurant claims that the mean mass of the packets of sauce is less than advertised.

Use the sample to test the fast food restaurant's claim using the 5% level of significance.

[7 marks]

10

Turn over ▶

8   The continuous random variable X has an exponential distribution with parameter λ

8 (a)   It is given that $P(X < m) = 0.5$

Show that $m < E(X)$

[3 marks]

8 (b) It is given that $P(X < s) = 0.25$ and $P(X > u) = 0.25$

Find $u - s$

Give your answer in the form $\dfrac{\ln k}{\lambda}$ where k is an integer.

[3 marks]

Answer _____

END OF QUESTIONS

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