



TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

PAPER 1

D513/01

2023

75 minutes

Additional materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but no extra paper is allowed.

You must complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.



1 Given that

$$\int_0^1 (ax + b) dx = 1$$

and

$$\int_0^1 x(ax + b) dx = 1$$

find the value of $a + b$.

A -1

B 0

C 1

D 2

E 3

F 4

G 5

- 2 The graphs of $y = x^2 + 5x + 6$ and $y = mx - 3$, where m is a constant, are plotted on the same set of axes.

Given that the graphs do not meet, what is the complete range of possible values of m ?

- A $-1 < m < 11$
- B $m < -1, m > 11$
- C $-\sqrt{11} < m < \sqrt{11}$
- D $m < -\sqrt{11}, m > \sqrt{11}$
- E $-11 < m < 1$
- F $m < -11, m > 1$

3 For any integer $n \geq 0$,

$$\int_n^{n+1} f(x) \, dx = n + 1$$

Evaluate

$$\int_0^3 f(x) \, dx + \int_1^3 f(x) \, dx + \int_2^3 f(x) \, dx + \int_4^3 f(x) \, dx + \int_5^3 f(x) \, dx$$

- A -2
- B 0
- C 1
- D 4
- E 18
- F 27

4 Evaluate

$$\sum_{n=0}^{\infty} \frac{\sin\left(n\pi + \frac{\pi}{3}\right)}{2^n}$$

A 0

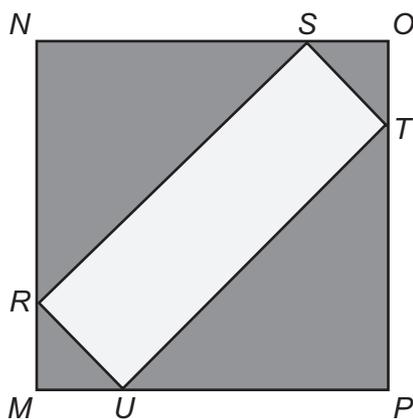
B $\frac{1}{3}$

C $\frac{\sqrt{3}}{3}$

D $\sqrt{3}$

E 3

- 5 The following shape has two lines of reflectional symmetry.



[diagram not to scale]

$MNOP$ is a square of perimeter 40 cm.

The vertices of rectangle $RSTU$ lie on the edge of square $MNOP$.

MR has length x cm.

What is the largest possible value of x such that $RSTU$ has area 20 cm^2 ?

- A $\sqrt{2}$
- B $\sqrt{10}$
- C $2\sqrt{15}$
- D $10\sqrt{2}$
- E $5 + \sqrt{5}$
- F $5 + \sqrt{15}$

6 In the simplified expansion of $(2+3x)^{12}$, how many of the terms have a coefficient that is divisible by 12?

A 0

B 2

C 5

D 10

E 11

F 12

G 13

7 $P(x)$ and $Q(x)$ are defined as follows:

$$P(x) = 2^x + 4$$

$$Q(x) = 2^{(2x-2)} - 2^{(x+2)} + 16$$

Find the largest value of x such that $P(x)$ and $Q(x)$ are in the ratio 4 : 1, respectively.

- A 5
- B 12
- C 32
- D $\log_2 3$
- E $\log_2 5$
- F $\log_2 12$
- G $\log_2 20$

8 A triangle XYZ is called *fun* if it has the following properties:

$$\text{angle } YXZ = 30^\circ$$

$$XY = \sqrt{3} a$$

$$YZ = a$$

where a is a constant.

For a given value of a , there are two distinct *fun* triangles S and T , where the area of S is greater than the area of T .

Find the ratio

area of S : area of T

- A 1:1
- B 2:1
- C $2:\sqrt{3}$
- D $\sqrt{3}:1$
- E 3:1

9 How many solutions are there to

$$(1 + 3 \cos 3\theta)^2 = 4$$

in the interval $0^\circ \leq \theta \leq 180^\circ$?

A 1

B 2

C 3

D 4

E 5

F 6

- 10 The trapezium rule with 4 strips is used to estimate the integral:

$$\int_{-2}^2 \sqrt{4-x^2} \, dx$$

What is the positive difference between the estimate and the exact value of the integral?

- A $2(\pi - 2 - 2\sqrt{3})$
- B $2(\pi - 1 - \sqrt{3})$
- C $2(2\pi - 1 - \sqrt{3})$
- D $4(\pi - 1 - \sqrt{3})$
- E $2\pi - 3\sqrt{3}$
- F $4\pi - 6\sqrt{3}$

11 It is given that $f(x) = x^2 - 6x$

The curves $y = f(kx)$ and $y = f(x - c)$ have the same minimum point, where $k > 0$ and $c > 0$

Which of the following is a correct expression for k in terms of c ?

A $k = \frac{3-c}{3}$

B $k = \frac{3}{c+3}$

C $k = \frac{c-6}{6}$

D $k = \frac{6}{6-c}$

E $k = \frac{c+9}{9}$

F $k = \frac{9}{c-9}$

12 How many solutions are there to the equation

$$\frac{2^{\tan^2 x}}{4^{\sin^2 x}} = 1$$

in the range $0 \leq x \leq 2\pi$?

A 2

B 3

C 4

D 5

E 6

F 7

G 8

13 Point P lies on the circle with equation $(x - 2)^2 + (y - 1)^2 = 16$

Point Q lies on the circle with equation $(x - 4)^2 + (y + 5)^2 = 16$

What is the maximum possible length of PQ ?

- A** 10
- B** 14
- C** 16
- D** $2\sqrt{34}$
- E** $10\sqrt{2}$
- F** $8 + 2\sqrt{10}$
- G** $16 + 2\sqrt{6}$

14 The function

$$f(x) = \frac{2}{3}x^3 + 2mx^2 + n, \quad m > 0$$

has three distinct real roots.

What is the complete range of possible values of n , in terms of m ?

A $-\frac{8}{3}m^3 < n < 0$

B $-\frac{4}{3}m^3 < n < 0$

C $0 < n < \frac{3}{2}m^2$

D $0 < n < \frac{40}{3}m^3$

E $n < -\frac{8}{3}m^3$

F $n < \frac{3}{2}m^2$

G $n > -\frac{4}{3}m^3$

H $n > \frac{40}{3}m^3$

- 15 The difference between the maximum and minimum values of the function $f(x) = a^{\cos x}$, where $a > 0$ and x is real, is 3.

Find the sum of the possible values of a .

- A 0
- B $\frac{3}{2}$
- C $\frac{5}{2}$
- D 3
- E $\sqrt{10}$
- F $\sqrt{13}$

16 A right-angled triangle has vertices at $(2, 3)$, $(9, -1)$ and $(5, k)$.

Find the sum of all the possible values of k .

A -8

B -6

C 0.25

D 2

E 2.25

F 8.25

G 10.25

17 A circle C_n is defined by

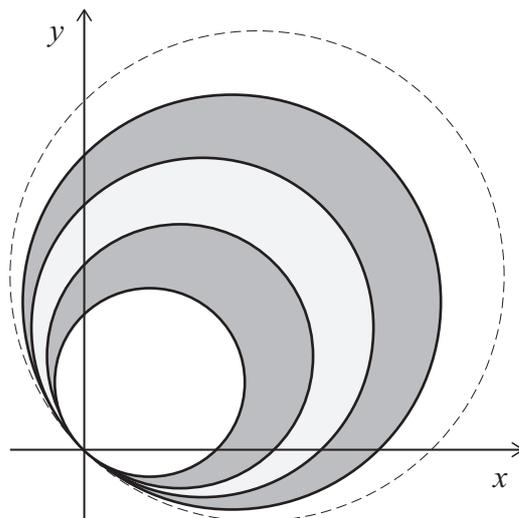
$$x^2 + y^2 = 2n(x + y)$$

where n is a positive integer.

C_1 and C_2 are drawn and the area between them is shaded.

Next, C_3 and C_4 are drawn and the area between them is shaded.

This is shown in the diagram.



[diagram not to scale]

This process continues until 100 circles have been drawn.

What is the total shaded area?

- A 100π
- B 500π
- C 2500π
- D 5050π
- E 10100π
- F 40400π

18 You are given that

$$S = 4 + \frac{8k}{7} + \frac{16k^2}{49} + \frac{32k^3}{343} + \dots + 4\left(\frac{2k}{7}\right)^n + \dots$$

The value for k is chosen as an integer in the range $-5 \leq k \leq 5$

All possible values for k are equally likely to be chosen.

What is the probability that the value of S is a finite number greater than 3?

A $\frac{1}{11}$

B $\frac{1}{10}$

C $\frac{3}{11}$

D $\frac{3}{10}$

E $\frac{5}{11}$

F $\frac{1}{2}$

G $\frac{7}{11}$

H $\frac{7}{10}$

19 The solution to the differential equation

$$\frac{dy}{dx} = |-6x| \quad \text{for all } x$$

is $y = f(x) + c$, where c is a constant.

Which one of the following is a correct expression for $f(x)$?

A $-\frac{6x}{|x|}$

B $\frac{6x}{|x|}$

C $-3x|x|$

D $3x|x|$

E $-3x^2$

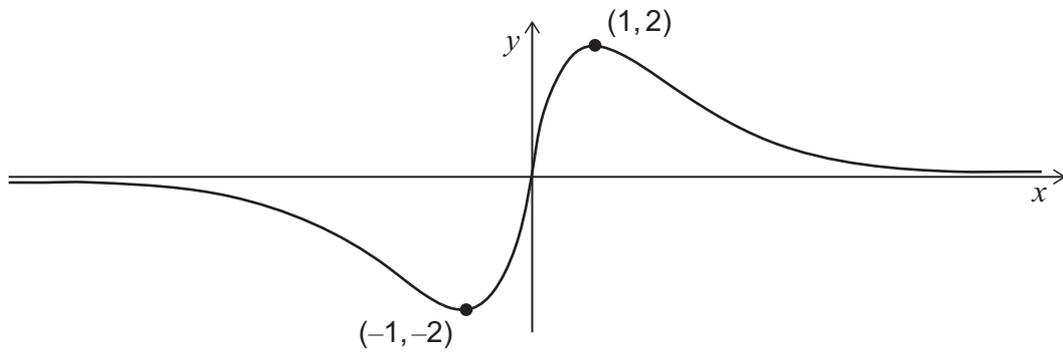
F $3x^2$

G $-x^3$

H x^3

20 The diagram shows the graph of $y = f(x)$

The function f attains its maximum value of 2 at $x = 1$, and its minimum value of -2 at $x = -1$



Find the difference between the maximum and minimum values of $(f(x))^2 - f(x)$

A 2

B $\frac{9}{4}$

C 4

D $\frac{17}{4}$

E 6

F $\frac{25}{4}$

G 8

H $\frac{33}{4}$