| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $\frac{1}{2}.\frac{\frac{4}{3}mg}{a}x^2$ | B1 | EPE term correct |
| | $\frac{1}{3}mgx$ | B1 | Work term correct |
| | Loss in KE = gain in EPE + work done against friction<br>$\frac{1}{2}mv^2 = \frac{1}{2}\times\frac{\frac{4}{3}mg}{a}x^2 + \frac{1}{3}mgx$ | M1 | Energy equation with 3 terms, allow sign error. |
| | $\frac{1}{2}\times\frac{1}{4}ga = \frac{\frac{2}{3}g}{a}x^2 + \frac{1}{3}gx$<br>$16x^2 + 8ax - 3a^2 = 0$<br>$(4x-a)(4x+3a) = 0$ | M1 | Obtain and attempt to solve a 3-term quadratic equation. |
| | $x = \frac{1}{4}a$ | A1 | |
| | | 5 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $m\frac{dv}{dt}=50-2v^2 \quad \frac{dv}{dt}=4\left(25-v^2\right)$ | **B1** | N2L |
| | $\frac{1}{10}\int\frac{1}{5-v}+\frac{1}{5+v}dv=\int 4dt$ | **M1** | Separate variables and use partial fractions. |
| | $\frac{1}{10}\left(-\ln(5-v)+\ln(5+v)\right)=4t+A$ | **M1 A1** | Integrate into log terms.<br>(Note: formula on MF19). |
| | Use $t=0, v=3$ to give $A=\frac{1}{10}\ln 4$ | **M1** | Use initial condition. |
| | $4t=\frac{1}{10}\ln\frac{5+v}{4(5-v)}$ leading to $\frac{5+v}{20-4v}=e^{40t}$ | **M1** | Rearrange to make $v$ the subject. |
| | $v=\frac{5\left(4-e^{-40t}\right)}{4+e^{-40t}}$ | **A1** | |
| | | **7** | |
| 4(b) | As $t\to\infty, v\to 5$ | **B1** | |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | [Mass is proportional to area]<br>\| \| Area \| Centre of mass from $AC$ \|<br>\| $ABC$ \| $\frac{1}{2}.6a.8a\ \ (=24a^2)$ \| $2a$ \|<br>\| $DEC$ \| $\frac{1}{2}x.5a$ \| $\frac{1}{3}x$ \|<br>\| $ADEB$ \| $24a^2 - \frac{5}{2}xa$ \| $\bar{x}$ \| | B1 | All correct for $ABC$ and $DEC$. |
| | Moments [about $AC$] $\bar{x}\left(24a^2 - \frac{5}{2}xa\right) = 24a^2 \times 2a - \frac{1}{3}x \times \frac{5}{2}ax$ | M1 | All moment terms present, dimensionally correct, allow sign error. |
| | | A1 | All correct moments about $AC$. |
| | $\bar{x} = \dfrac{288a^2 - 5x^2}{3(48a - 5x)}$ | A1 | AEF |
| | | 4 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(b) | On the point of toppling about *E*: $\bar{x} = x$, $\dfrac{288a^2 - 5x^2}{3(48a - 5x)} = x$ | **B1 FT** | FT *their* expression for $\bar{x}$ from part **(a)**. |
| | Rearrange to 3-term quadratic: $10x^2 - 144ax + 288a^2 = 0$ | **M1** | Allow 3-term inequality. |
| | $2(5x - 12a)(x - 12a) = 0$,<br>$x = \dfrac{12}{5}a$ | **A1** | Single correct answer, no inequality, CWO. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Angular speeds of $P$ and $Q$ are equal, so $\frac{v_Q}{x} = \frac{v_P}{3a - x}$ | M1 | |
| | $v_Q = \frac{2x\sqrt{ag}}{3a - x}$ | A1 | Shown convincingly: angular speeds equal stated. AG |
| | | 2 | |
| 6(b) | For $P$: $T + mg\cos 60° = \frac{m \times 4ag}{3a - x}$ | B1 | |
| | For $Q$: $T - mg\cos 60° = \frac{mv_Q^2}{x}$ | B1 | |
| | Eliminate $T$: $-mg\cos 60° + \frac{m.4ag}{3a - x} = mg\cos 60° + \frac{mv_Q^2}{x}$ | M1 | |
| | $\frac{m \times 4ag}{3a - x} = 1 + \frac{mx4ag}{(3a - x)^2}$<br>$4a(3a - x) = (3a - x)^2 + 4ax$, $x^2 + 2ax - 3a^2 = 0$ | M1 | Solve to find $x$.<br>Obtain 3-term quadratic equation. |
| | $(x - a)(x + 3a) = 0$, $x = a$ | A1 | |
| | | 5 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | Energy changes from initial position:<br>Gain in KE of $P$: $\frac{1}{2}m\left(4ag-u^2\right)$ | B1 | KEs correct. |
| | Loss in KE of $Q$: $\frac{1}{2}m\left(\left(\frac{u}{2}\right)^2-v_Q^2\right)$<br>Loss in GPE of $P = mg(3a-x)(1-\cos 60°)\ \ (=mga)$<br>Gain in GPE of $Q = mgx(1-\cos 60°)\ \ \left(=\frac{1}{2}mga\right)$ | B1FT | GPEs correct. |
| | $\frac{1}{2}m\left(4ag-u^2\right)-\frac{1}{2}m\left(\left(\frac{u}{2}\right)^2-v_Q^2\right)=-mgx(1-\cos 60°)+mg(3a-x)(1-\cos 60°)$ | M1 | Energy equation. |
| | Simplify: $4ag-\frac{5}{4}u^2+ag=ag$<br>$u^2=\frac{16}{5}ag,\quad u=\frac{4}{5}\sqrt{5ag}$ | A1 | AEF |
| | | 4 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Use F = ma: $20 = \frac{2 \times v^2}{0.6}$ OR $20 = 2 \times 0.6\omega^2$ | **M1** | |
| | $v^2 = 6$ OR $\omega^2 = \frac{50}{3}$ | **A1** | |
| | Number of revolutions per min = $\frac{60v}{0.6 \times 2\pi}$ OR $\frac{60\omega}{2\pi}$<br>so 39(.0) revolutions | **A1 FT** | 38.9848…. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | Loss in KE = Gain in EPE, so<br>$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{2} \times \frac{4mg}{a}\left(\left(\frac{1}{2}a\right)^2 - \left(\frac{1}{4}a\right)^2\right)$ | **B1** | EPE terms correct. |
| | | **M1** | All 4 terms and no extras. |
| | $\frac{3}{4}mv^2 = \frac{4mg}{a} \times \frac{3}{16}a^2$ | **M1** | Simplify. |
| | $v^2 = ag, \quad v = \sqrt{ag}$ | **A1** | |
| | | **4** | |
| 2(b) | Hooke's law: tension = $\frac{4mg}{a} \times \frac{1}{2}a$ (= $2mg$) | **M1** | |
| | Acceleration = $\frac{2mg}{m} = 2g$ | **A1** | Accept $-2g$. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Let speeds of $A$ and $B$ along line of centres after collision be $V_A$ and $V_B$<br>$V_A + V_B = u\cos 30°$ (1) | **M1** | Allow sign errors, allow missing $m$. |
| | $-V_A + V_B = eu\cos 30°$ (2) | **M1** | Signs on LHS must be consistent with (1). |
| | Speeds perpendicular to line of centres after collision are $u\sin 30°$ and $2u$<br>Moving in same direction, so $\frac{V_A}{u\sin 30°} = \frac{V_B}{2u}$ (3) | **B1** | SOI<br>$V_B = 4V_A$ |
| | Use $V_B = 4V_A$ in (1): $5V_A = u\cos 30°$<br>From (2): $3V_A = eu\cos 30°$ then<br>Combine to find equation in $e$ only. | **M1** | A complete method to find equation in $e$ only |
| | $e = \frac{3}{5}$ | **A1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | **Alternative method for question 4(a)** | | |
| | Let speeds of $A$ and $B$ along line of centres after collision be $V_A$ and $V_B$<br>$V_A + V_B = u\cos 30°$ (1) | M1 | Allow sign errors, allow missing $m$. |
| | $-V_A + V_B = eu\cos 30°$ (2) | M1 | Signs on LHS must be consistent with (1). |
| | Speeds perpendicular to line of centres after collision are $u\sin 30°$ and $2u$<br>Moving in same direction, so $\frac{V_A}{u\sin 30°} = \frac{V_B}{2u}$ (3) | B1 | SOI<br>$V_B = 4V_A$ |
| | Solve (1) and (2): $V_A = \frac{1}{2}u(1-e)\cos 30°$, $V_B = \frac{1}{2}u(1+e)\cos 30°$<br>Substitute in (3) to find equation in $e$ only . | M1 | Note: $V_A = \frac{u}{10}\sqrt{3}$, $V_B = \frac{4u}{10}\sqrt{3}$ |
| | $e = \frac{3}{5}$ | A1 | |
| | | **5** | |
| 4(b) | KE after $= \frac{1}{2}m\left(V_A^2 + \left(\frac{u}{2}\right)^2\right) + \frac{1}{2}m\ ((2u)^2 + V_B^2)$ | B1 | Correct expression for KE for one of the spheres, after collision, with both components. |
| | KE for A after $= \frac{7}{50}mu^2$ or KE for B after $= \frac{56}{25}mu^2$<br>or KE loss for A $= \frac{9}{25}mu^2$ or KE **gain** for B $= \frac{6}{25}mu^2$ | B1 | Implied by total KE after $= \frac{119}{50}mu^2$ . |
| | Total loss in KE $= \frac{3}{25}mu^2$ | B1 | Term $\frac{1}{2}m(2u)^2$ may be omitted from KE of $B$ before and after. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Let $N$ be normal reaction at $B$ and $F$ the frictional force acting downwards<br>$\uparrow T\cos\theta = F + W$ | B1 | |
| | $\rightarrow T\sin\theta = N$ | B1 | |
| | Moments about $B$: $Ta = W\sin\theta \times a + W\cos\theta \times a$ | M1A1 | A moments equation with all relevant forces. |
| | $F = \frac{1}{2}N$ used | M1 | |
| | $(\cos\theta + \sin\theta)\left(\cos\theta - \frac{1}{2}\sin\theta\right) = 1$ oe | M1 | Combine to obtain equation in $\theta$.<br>Equation in trigonometric functions only. |
| | $\frac{1}{2}\cos\theta\sin\theta = \frac{3}{2}(\sin\theta)^2$<br>$\sin\theta(\cos\theta - 3\sin\theta) = 0$ | M1 | Solve trigonometric equation. |
| | $\tan\theta = \frac{1}{3}$ | A1 | |
| | | 8 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $T = 4mg.\frac{ka}{a}$ | B1 | Use Hooke's law |
| | $T\sin\theta = \left(\frac{mrg}{a}\right) = m(k+1)a\sin\theta.\frac{g}{a}$ | M1 | N2L horizontally. Must see $T$ and $k$. |
| | $T = mg(k+1)$ | A1 | |
| | Equate: $k = \frac{1}{3}$ | A1 | |
| | | 4 | |
| 3(b) | $\uparrow T\cos\theta = mg$ | M1 | |
| | $(T = \frac{4}{3}mg)$ $\cos\theta = \frac{mg}{\frac{4}{3}mg} = \frac{3}{4}$ | A1 | |
| | | 2 | |