2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion. [5]

4 A particle of mass 0.5 kg moves along a horizontal straight line. Its velocity is $v\,\text{m}\,\text{s}^{-1}$ at time t s. The forces acting on the particle are a driving force of magnitude 50 N and a resistance of magnitude $2v^2$ N. The initial velocity of the particle is $3\,\text{m}\,\text{s}^{-1}$.

(a) Find an expression for v in terms of t. [7]

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(b) Deduce the limiting value of v. [1]

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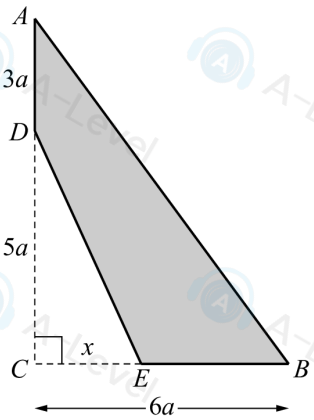

A uniform lamina is in the form of a triangle ABC, with $AC = 8a$, $BC = 6a$ and angle $ACB = 90°$. The point D on AC is such that $AD = 3a$. The point E on CB is such that $CE = x$ (see diagram). The triangle CDE is removed from the lamina.

(a) Find, in terms of a and x, the distance of the centre of mass of the remaining object $ADEB$ from AC. [4]

The object $ADEB$ is on the point of toppling about the point E when the object is in the vertical plane with its edge EB on a smooth horizontal surface.

(b) Find x in terms of a. [3]

6 A particle P of mass m is attached to one end of a light inextensible rod of length $3a$. An identical particle Q is attached to the other end of the rod. The rod is smoothly pivoted at a point O on the rod, where $OQ = x$. The system, of rod and particles, rotates about O in a vertical plane.

At an instant when the rod is vertical, with P above Q, the particle P is moving horizontally with speed u. When the rod has turned through an angle of 60° from the vertical, the speed of P is $2\sqrt{ag}$, and the tensions in the two parts of the rod, OP and OQ, have equal magnitudes.

(a) Show that the speed of Q when the rod has turned through an angle of 60° from the vertical is $\frac{2x}{3a-x}\sqrt{ag}$. [2]

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(b) Find x in terms of a. [5]

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(c) Find u in terms of a and g. [4]

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2 A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is fixed to a point O on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected along the surface in the direction OP. When the length of the string is $\frac{5}{4}a$, the speed of P is v. When the length of the string is $\frac{3}{2}a$, the speed of P is $\frac{1}{2}v$.

(a) Find an expression for v in terms of a and g. [4]

(b) Find, in terms of g, the acceleration of P when the stretched length of the string is $\frac{3}{2}a$. [2]

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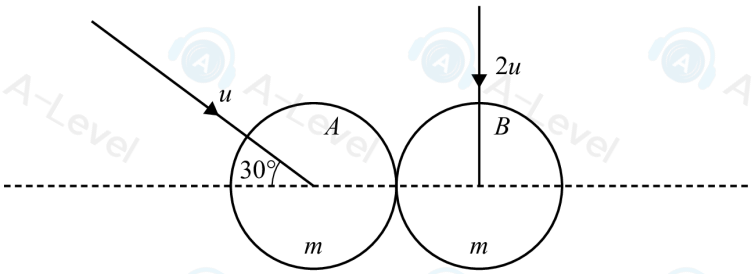
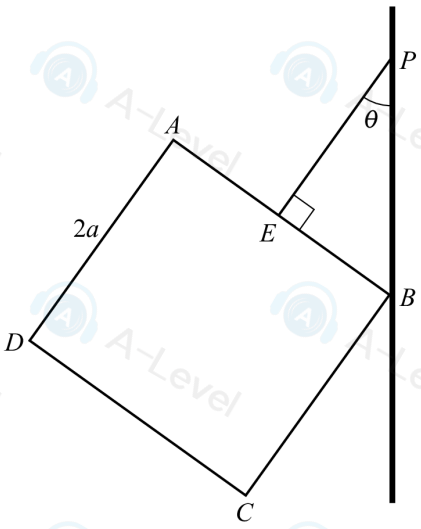


Two identical smooth uniform spheres A and B each have mass m. The two spheres are moving on a smooth horizontal surface when they collide with speeds u and $2u$ respectively. Immediately before the collision, A's direction of motion makes an angle of 30° with the line of centres, and B's direction of motion is perpendicular to the line of centres (see diagram). After the collision, A and B are moving in the same direction. The coefficient of restitution between the spheres is e.

(a) Find the value of e. [5]

(b) Find the loss in the total kinetic energy of the spheres as a result of the collision. [3]

3



A uniform square lamina of side $2a$ and weight W is suspended from a light inextensible string attached to the midpoint E of the side AB. The other end of the string is attached to a fixed point P on a rough vertical wall. The vertex B of the lamina is in contact with the wall. The string EP is perpendicular to the side AB and makes an angle θ with the wall (see diagram). The string and the lamina are in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the lamina is $\frac{1}{2}$.

Given that the vertex B is about to slip up the wall, find the value of $\tan\theta$. [8]

3 One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O. The length of the string during this motion is $(k+1)a$.

(a) Find the value of k. [4]

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(b) Find the value of $\cos\theta$. [2]

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[Turn over