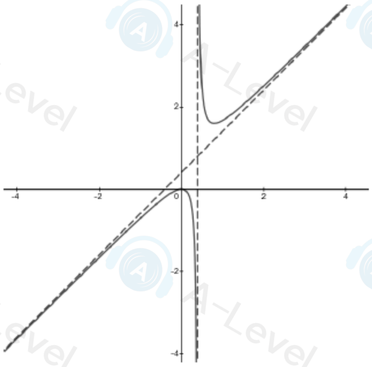
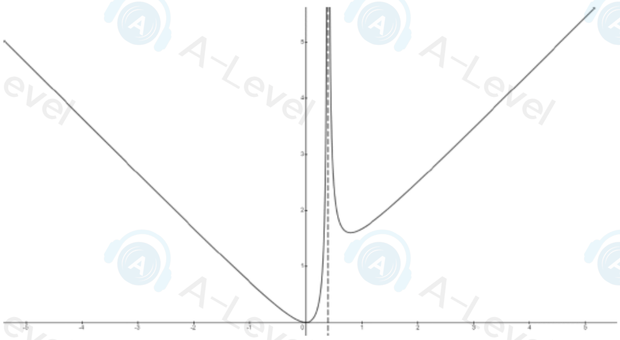
Question	Answer	Marks	Guidance
7(a)	$x=\frac{2}{5}$	B1	Vertical asymptote.
	$y=\dfrac{(5x-2)(x+\frac{2}{5})+\frac{4}{5}}{5x-2}$ leading to $y=x+\frac{2}{5}$	M1 A1	Oblique asymptote.
		3	
7(b)	$\dfrac{dy}{dx}=\dfrac{(5x-2)(10x)-(5x^2)(5)}{(5x-2)^2}$	M1	Finds $\dfrac{dy}{dx}$.
	$5x^2-4x=0$	M1	Sets equal to 0 and forms quadratic equation.
	$(0,0)$, $\left(\frac{4}{5},\frac{8}{5}\right)$	A1 A1	
		4	
7(c)		B1	Axes and asymptotes.
		B1	Correct upper branch and asymptotic behaviour.
		B1	Correct lower branch.
		3	

Question	Answer	Marks	Guidance
7(d)		B1 FT	FT from sketch in part **(c)**.
		B1	Correct shape as x tends to infinity.
	$\dfrac{5x^2}{5x-2}=2$ or $\dfrac{5x^2}{5x-2}=-2$ $5x^2-10x+4=0$ or $5x^2+10x-4=0$	M2	Finds critical points, award M1 for each case.
	$x=-1-\frac{3}{5}\sqrt{5}$, $x=-1+\frac{3}{5}\sqrt{5}$ or $x=1-\frac{1}{5}\sqrt{5}$, $x=1+\frac{1}{5}\sqrt{5}$	A1	Must be exact.
	$-1-\frac{3}{5}\sqrt{5}<x<-1+\frac{3}{5}\sqrt{5}$, $1-\frac{1}{5}\sqrt{5}<x<1+\frac{1}{5}\sqrt{5}$	A1 FT	Follow through on use of decimals.
		6	

Question	Answer	Marks	Guidance
7(a)	$\overrightarrow{AB} = 4\mathbf{i}-\mathbf{j}+\mathbf{k}$ $\quad \overrightarrow{CD} = \mathbf{j}+(\lambda-3)\mathbf{k}$	B1	Finds direction vectors of the two lines.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ 0 & 1 & \lambda-3 \end{vmatrix} = \begin{pmatrix} 2-\lambda \\ 12-4\lambda \\ 4 \end{pmatrix}$	M1 A1	Finds common perpendicular. This may also be done by setting up simultaneous equations and solving them.
	$\left\lvert \dfrac{1}{\sqrt{(\lambda-2)^2+(4\lambda-12)^2+16}} \begin{pmatrix} -5 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2-\lambda \\ 12-4\lambda \\ 4 \end{pmatrix} \right\rvert$	M1 A1	Uses formula for perpendicular distance.
	$\left\lvert \dfrac{30-3\lambda}{\sqrt{17\lambda^2-100\lambda+164}} \right\rvert = 3 \Rightarrow 9(\lambda-10)^2 = 9\left(17\lambda^2-100\lambda+164\right)$	M1	Sets equal to 3 and forms in quadratic in λ.
	$16\lambda^2-80\lambda+64=0$ leading to $\lambda^2-5\lambda+4=0$	A1	AG
		7	
7(b)(i)	$\mathbf{r} = 7\mathbf{i}+4\mathbf{j}-\mathbf{k}+s(4\mathbf{i}-\mathbf{j}+\mathbf{k})+t(-5\mathbf{i}+3\mathbf{j}+2\mathbf{k})$	M1 A1	OE M1 for using a correct point and attempting to find relevant direction vectors.
		2	
7(b)(ii)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ -5 & 3 & 5 \end{vmatrix} = \begin{pmatrix} -8 \\ -25 \\ 7 \end{pmatrix}$	M1 A1	Finds normal to the plane Π_2.
	$-8(7)-25(4)+7(-1)$ leading to $-8x-25y+7z=-163$	M1 A1	Substitutes point.
		4	

Question	Answer	Marks	Guidance
7(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{vmatrix} = \begin{pmatrix} -5 \\ -13 \\ 7 \end{pmatrix}$	M1 A1	Finds normal to the plane Π_1.
	$\begin{pmatrix} -5 \\ -13 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ -25 \\ 7 \end{pmatrix} = \sqrt{243}\sqrt{738}\cos\theta$ leading to $\cos\theta = \dfrac{414}{\sqrt{243}\sqrt{738}}$	M1 A1	Uses dot product of normal vectors. $\cos = 0.9776176...$
	12.1°	A1	Mark final answer. Accept 0.212^c
		5	

Question	Answer	Marks	Guidance
1(a)	$k\begin{vmatrix} 5 & 2 \\ 3 & -k \end{vmatrix} - \begin{vmatrix} 6 & 2 \\ -1 & -k \end{vmatrix} = k(-5k-6)-(-6k+2) = -5k^2-2$	M1 A1	Evaluates determinant, forms quadratic expression. (Allow 1 slip in the calculation of the determinant for M1 only)
	No real value of $k \Rightarrow$ Non-singular	A1	Convincing conclusion using the discriminant or determinant.
		3	
1(b)	$3k+1=1$ or $-\frac{1}{2} = -\dfrac{1}{5k^2+2}$	M1	Uses $\mathbf{AA}^{-1}=\mathbf{I}$ or $\det(\mathbf{A}^{-1}) = (\det \mathbf{A})^{-1}$ to find equation in k. Could also use a minor determinant e.g. $-k\times1-3\times0=0$
	$k=0$	A1	Only $k=0$, A0 for any additional solutions.
		2	

Question	Answer	Marks	Guidance
5(c)	$-\mathbf{i}-\mathbf{j}+\mathbf{k}$	B1	Finding direction between the two given points.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 1 \\ 1 & 4 & -3 \end{vmatrix} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$	M1	Find common perpendicular. Allow use of their normal from part **(b)**.
	$x+2y+3z=6$	A1	OE
		3	

Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -1 \\ 3 & 2 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix}$	M1 A1	Finds vector perpendicular to Π.
	$6(2)-(3)+8(-2)=-7$	M1	Substitutes point on Π.
	$6x-y+8z=-7$	A1	
		4	

Question	Answer	Marks	Guidance
5(b)	$\overrightarrow{OF}=\overrightarrow{OP}+\overrightarrow{PF}=\begin{pmatrix} 4 \\ 2 \\ 9 \end{pmatrix}+t\begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix}=\begin{pmatrix} 4+6t \\ 2-t \\ 9+8t \end{pmatrix}$	M1 A1FT	Uses $\overrightarrow{OP}$ and multiple of their normal to Π.
	$6(4+6t)-(2-t)+8(9+8t)=-7 \Rightarrow 101t+94=-7$	M1	Substitutes into the equation for Π.
	$t=-1 \Rightarrow \overrightarrow{OF}=\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$	A1	
		4	
5(c)	$\begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix}=\sqrt{35}\sqrt{101}\cos\alpha \Rightarrow \cos\alpha=\frac{5}{\sqrt{35}\sqrt{101}}$	M1 A1FT	Uses dot product of $3\mathbf{i}+5\mathbf{j}-\mathbf{k}$ and their normal in a correct formula.
	Acute angle between l and Π is $90-\alpha=4.8^\circ$	A1	Mark final answer. No ISW
		3	

Question	Answer	Marks	Guidance
7(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix} = \begin{pmatrix} 6 \\ -3 \\ -9 \end{pmatrix} \sim \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$	M1 A1	Finds common perpendicular. Allow one error.
	$-2(1)+(3)+3(-2)=-5$	M1	Substitutes point on l_1.
	$2x-y-3z=5$	A1	CAO.
		4	

Question	Answer	Marks	Guidance
7(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 1 & 1 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix}$	M1 A1	Finds the normal to Π_2.
	$\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix}=\sqrt{14}\sqrt{77}\cos\theta \Rightarrow \cos\theta=\frac{-7}{\sqrt{14}\sqrt{77}}$	M1	Uses dot product of normal vectors.
	77.7°	A1	No ISW. Accept 1.36 rad.
		4	

Question	Answer	Marks	Guidance
7(c)	$\overrightarrow{OP}=\begin{pmatrix}1+2\lambda\\3+\lambda\\-2+\lambda\end{pmatrix},\overrightarrow{OQ}=\begin{pmatrix}1+\mu\\-2-4\mu\\9+2\mu\end{pmatrix}\Rightarrow\overrightarrow{PQ}=\begin{pmatrix}\mu-2\lambda\\-5-4\mu-\lambda\\11+2\mu-\lambda\end{pmatrix}$	M1 A1	Finds $\overrightarrow{PQ}$.
	$\begin{pmatrix}\mu-2\lambda\\-5-4\mu-\lambda\\11+2\mu-\lambda\end{pmatrix}\bullet\begin{pmatrix}2\\1\\1\end{pmatrix}=0$ or $\begin{pmatrix}\mu-2\lambda\\-5-4\mu-\lambda\\11+2\mu-\lambda\end{pmatrix}=k\begin{pmatrix}6\\-3\\-9\end{pmatrix}$	M1	Uses that dot product of $\overrightarrow{PQ}$ with line direction is zero, or, alternatively, $\overrightarrow{PQ}$ is a multiple of the common perpendicular.
	$-6\lambda+6=0$	A1	Deduces one equation.
	$\begin{pmatrix}\mu-2\lambda\\-5-4\mu-\lambda\\11+2\mu-\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\-4\\2\end{pmatrix}=0\Rightarrow 21\mu+42=0$	A1	Deduces second equation.
	$\lambda=1\Rightarrow\overrightarrow{OP}=\begin{pmatrix}3\\4\\-1\end{pmatrix}$ or $\mu=-2\Rightarrow\overrightarrow{OQ}=\begin{pmatrix}-1\\6\\5\end{pmatrix}$	M1	Solves for λ or μ and substitutes into $\overrightarrow{OP}$ or $\overrightarrow{OQ}$
	$\mathbf{r}=\begin{pmatrix}-1\\6\\5\end{pmatrix}+t\begin{pmatrix}6\\-3\\-9\end{pmatrix}$	A1 FT	OE. FT using their common perpendicular. Must have "$\mathbf{r}=$".
		7	

Question	Answer	Marks	Guidance
4(a)	$\begin{pmatrix}2\\-2\\3\end{pmatrix}-\begin{pmatrix}-2\\-3\\-5\end{pmatrix}=\begin{pmatrix}4\\1\\8\end{pmatrix}$	B1	Finds direction of one line to another.
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-4&3&5\\2&-3&1\end{vmatrix}=\begin{pmatrix}18\\14\\6\end{pmatrix}\sim\begin{pmatrix}9\\7\\3\end{pmatrix}$	M1 A1	Find common perpendicular.
	$\frac{1}{\sqrt{139}}\left\lvert\begin{pmatrix}4\\1\\8\end{pmatrix}\bullet\begin{pmatrix}9\\7\\3\end{pmatrix}\right\rvert=\frac{67}{\sqrt{139}}(=5.68)$	M1 A1	Uses formula for shortest distance.
		5	
4(b)	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&0&1\\-4&3&5\end{vmatrix}=\begin{pmatrix}3\\9\\-3\end{pmatrix}\sim\begin{pmatrix}1\\3\\-1\end{pmatrix}$	M1 A1	Finds vector perpendicular to the plane.
	$1(-1)+3(-3)-1(-4)=-6\Rightarrow x+3y-z=-6$	M1 A1	Uses point in the plane.
		4	

Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 2 & 3 & 0 \end{vmatrix} = \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}$	M1 A1	Finds vector perpendicular to Π.
	$-3(-2)+2(3)+3(3)=21$	M1	Substitutes point on Π.
	$-3x+2y+3z=21$	A1	
		4	
5(b)	$\begin{pmatrix} 2 \\ -3 \\ 5+t \end{pmatrix}$	B1	Forms general point on line (given as a single vector).
	$-3(2)+2(-3)+3(5+t)=21$ leading to $t=6$	M1	Substitutes into the equation for Π.
	$\begin{pmatrix} 2 \\ -3 \\ 11 \end{pmatrix}$	A1	
		3	
5(c)	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix} = \sqrt{1}\sqrt{22}\cos\alpha$ leading to $\cos\alpha = \frac{3}{\sqrt{22}}$	M1 A1 FT	Uses dot product of $\mathbf{k}$ and their normal.
	Acute angle between l and Π is $90-\alpha = 39.8^\circ$	A1	
		3	

Question	Answer	Marks	Guidance
5(d)	$\begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$	B1	Finds direction vector from P to plane.
	$\frac{1}{\sqrt{22}} \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix} = \frac{18}{\sqrt{22}} = 3.84$	M1 A1	Uses dot product of their direction and normal vectors.
		3	