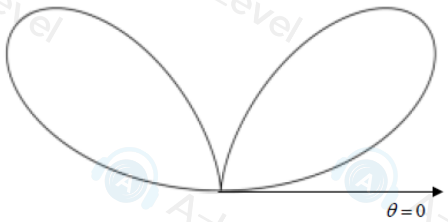
Question	Answer	Marks	Guidance
7(a)		B1	One loop correct with initial line.
		B1	Second loop correct with correct form at the pole.
	$\theta = \frac{1}{2}\pi$	B1	May be seen on their diagram.
		3	
7(b)	$r^5 = 2(r\sin\theta)(r\cos\theta)^2$	M1	Use of $\sin 2\theta = 2\sin\theta\cos\theta$, and $x = r\cos\theta$ or $y = r\sin\theta$.
	$r^5 = 2yx^2$	A1	
	$\left(x^2+y^2\right)^{\frac{5}{2}} = 2x^2 y$	A1	AEF
		3	

Question	Answer	Marks	Guidance
7(c)	$\frac{1}{2}\int_0^{\pi} \sin(2\theta)\cos\theta \, d\theta$	M1	Applies $\frac{1}{2}\int r^2 \, d\theta$.
	$\frac{1}{2}\int \sin(2\theta)\cos\theta \, d\theta$	M1	Attempt to integrate in a valid way. May apply $\sin(2\theta) = 2\sin\theta\cos\theta$.
	$= \int \sin\theta\cos^2\theta \, d\theta = -\frac{1}{3}\cos^3\theta + [c]$	A1	Correct answer (there are alternative forms).
	$= \left[-\frac{1}{3}\cos^3\theta\right]_0^{\pi} = \frac{2}{3}$	A1	CAO
		4	
7(d)	$\frac{dr}{d\theta} = \frac{1}{2}\left(\sin 2\theta\cos\theta\right)^{-\frac{1}{2}}\left(2\cos 2\theta\cos\theta - \sin 2\theta\sin\theta\right)$	*M1 A1	Differentiates with respect to θ.
	$2\cos 2\theta\cos\theta - \sin 2\theta\sin\theta = 6\cos^3\theta - 4\cos\theta$	dM1	Correct use of relevant identities to express in terms of a single trig function.
	$6\cos^3\theta - 4\cos\theta = 0 \Rightarrow \cos^2\theta = \frac{2}{3}$	dM1 A1	Sets derivative equal to 0 and solves to find one of $\sin^2\theta = \frac{1}{3}$, $\cos^2\theta = \frac{2}{3}$, $\tan^2\theta = \frac{1}{2}$ or θ=0.6155
	$r = \sqrt{\frac{4}{3\sqrt{3}}} = \sqrt{\frac{4\sqrt{3}}{9}} = 0.877$	A1	

Question	Answer	Marks	Guidance
7(d)	**Alternative method for question 7(d)**		
	$2r\frac{dr}{d\theta} = 2\left(-2\sin^2\theta\cos\theta + \cos^3\theta\right)$	*M1 A1	Differentiates [RHS] with respect to θ.
	$2\left(-2\left(1-\cos^2\theta\right)\cos\theta + \cos^3\theta\right) = 6\cos^3\theta - 4\cos\theta$	dM1	Correct use of relevant identities in terms of a single trig function
	$6\cos^3\theta - 4\cos\theta = 0 \Rightarrow \cos^2\theta = \frac{2}{3}$	dM1 A1	Sets derivative equal to 0 and solves to find one of $\sin^2\theta = \frac{1}{3}$, $\cos^2\theta = \frac{2}{3}$, $\tan^2\theta = \frac{1}{2}$ or θ=0.6155
	$r = \sqrt{\frac{4}{3\sqrt{3}}} = \sqrt{\frac{4\sqrt{3}}{9}} = 0.877$	A1	
		6	

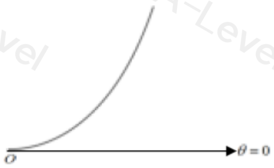
Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 1 & 3 & 1 \end{vmatrix} = \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$	M1 A1	Finds perpendicular to Π_1.
	$5(-3) - 2(-1) + (-1) = -14$	M1	Uses point on Π_1.
	$5x - 2y + z = -14$	A1	
		4	

Question	Answer	Marks	Guidance
5(b)	$\frac{1}{\sqrt{5^2+2^2+1^2}} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix}$	M1 A1	Uses correct formula for perpendicular distance from point to Π_1.
	$\frac{7}{\sqrt{30}}$	A1	Accept 1.28.
		3	
5(c)	States point common to both planes e.g. $\begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$.	B1	Or $\begin{pmatrix} 0 \\ 0 \\ -14 \end{pmatrix}$.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 8 \\ 16 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$	M1 A1	Finds direction of line.
	$r = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$	A1	OE
		4	

Question	Answer	Marks	Guidance
7(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix} = \begin{pmatrix} 6 \\ -3 \\ -9 \end{pmatrix} \sim \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$	M1 A1	Finds common perpendicular. Allow one error.
	$-2(1) + (3) + 3(-2) = -5$	M1	Substitutes point on l_1.
	$2x - y - 3z = 5$	A1	CAO.
		4	

Question	Answer	Marks	Guidance
7(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 1 & 1 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix}$	M1 A1	Finds the normal to Π_2.
	$\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix} = \sqrt{14}\sqrt{77}\cos\theta \Rightarrow \cos\theta = \frac{-7}{\sqrt{14}\sqrt{77}}$	M1	Uses dot product of normal vectors.
	77.7°	A1	No ISW. Accept 1.36 rad.
		4	

Question	Answer	Marks	Guidance
7(c)	$\overrightarrow{OP}=\begin{pmatrix}1+2\lambda\\3+\lambda\\-2+\lambda\end{pmatrix},\overrightarrow{OQ}=\begin{pmatrix}1+\mu\\-2-4\mu\\9+2\mu\end{pmatrix}\Rightarrow\overrightarrow{PQ}=\begin{pmatrix}\mu-2\lambda\\-5-4\mu-\lambda\\11+2\mu-\lambda\end{pmatrix}$	M1 A1	Finds $\overrightarrow{PQ}$.
	$\begin{pmatrix}\mu-2\lambda\\-5-4\mu-\lambda\\11+2\mu-\lambda\end{pmatrix}\cdot\begin{pmatrix}2\\1\\1\end{pmatrix}=0$ or $\begin{pmatrix}\mu-2\lambda\\-5-4\mu-\lambda\\11+2\mu-\lambda\end{pmatrix}=k\begin{pmatrix}6\\-3\\-9\end{pmatrix}$	M1	Uses that dot product of $\overrightarrow{PQ}$ with line direction is zero, or, alternatively, $\overrightarrow{PQ}$ is a multiple of the common perpendicular.
	$-6\lambda+6=0$	A1	Deduces one equation.
	$\begin{pmatrix}\mu-2\lambda\\-5-4\mu-\lambda\\11+2\mu-\lambda\end{pmatrix}\cdot\begin{pmatrix}1\\-4\\2\end{pmatrix}=0\Rightarrow 21\mu+42=0$	A1	Deduces second equation.
	$\lambda=1\Rightarrow\overrightarrow{OP}=\begin{pmatrix}3\\4\\-1\end{pmatrix}$ or $\mu=-2\Rightarrow\overrightarrow{OQ}=\begin{pmatrix}-1\\6\\5\end{pmatrix}$	M1	Solves for λ or μ and substitutes into $\overrightarrow{OP}$ or $\overrightarrow{OQ}$
	$\mathbf{r}=\begin{pmatrix}-1\\6\\5\end{pmatrix}+t\begin{pmatrix}6\\-3\\-9\end{pmatrix}$	A1 FT	OE. FT using their common perpendicular. Must have "$\mathbf{r}=$".
		7	

Question	Answer	Marks
5(a)		B1
	Maximum distance of C from the pole is a.	B1
		2
5(b)	$\frac{1}{2}a^2\int_0^{\frac{1}{4}\pi}\tan^2\theta\,d\theta$	M1
	$=\frac{1}{2}a^2\int_0^{\frac{1}{4}\pi}\sec^2\theta-1\,d\theta=\frac{1}{2}a^2\left[\tan\theta-\theta\right]_0^{\frac{1}{4}\pi}$	M1 A1
	$=\frac{1}{2}a^2\left(1-\frac{1}{4}\pi\right)$	A1
		4
5(c)	$\sqrt{x^2+y^2}=a\frac{y}{x}$	B1
	$x^2\left(x^2+y^2\right)=a^2y^2\Rightarrow y^2\left(a^2-x^2\right)=x^4$	M1
	$y=\frac{x^2}{\sqrt{\left(a^2-x^2\right)}}$ **AG**	A1
		3

Question	Answer	Marks
5(d)	$\frac{1}{2}\left(a\cos\frac{1}{4}\pi\right)\left(a\sin\frac{1}{4}\pi\right)-\frac{1}{2}a^2\left(1-\frac{1}{4}\pi\right)=\frac{1}{4}a^2\left(\frac{1}{2}\pi-1\right)$ (**A1 FT** *their* **(b)**)	M1 A1 FT
		11

Question	Answer	Marks	Guidance
1(a)	$\begin{pmatrix}14 & 0\\ 0 & 1\end{pmatrix}$	B1	Stretch parallel to the x-axis, scale factor k $(k \neq 0)$.
	$\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$	B1	Rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.
	$\mathbf{M}=\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}14 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}7 & -\frac{\sqrt{3}}{2}\\ 7\sqrt{3} & \frac{1}{2}\end{pmatrix}$	M1	Correct order.
	$2\mathbf{M}=\begin{pmatrix}14 & -\sqrt{3}\\ 14\sqrt{3} & 1\end{pmatrix}$.	A1	AG
		4	

Question	Answer	Marks	Guidance
1(b)	$\begin{pmatrix}7 & -\frac{\sqrt{3}}{2}\\ 7\sqrt{3} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}7x-\frac{\sqrt{3}}{2}y\\ 7\sqrt{3}x+\frac{1}{2}y\end{pmatrix}$	B1	Transforms $\begin{pmatrix}x\\ y\end{pmatrix}$ to $\begin{pmatrix}X\\ Y\end{pmatrix}$
	$7\sqrt{3}x+\frac{1}{2}mx=m\left(7x-\frac{\sqrt{3}}{2}mx\right)$ $7\sqrt{3}+\frac{1}{2}m=7m-\frac{\sqrt{3}}{2}m^2$	M1	Uses $y=mx$ and $Y=mX$.
	$\sqrt{3}m^2-13m+14\sqrt{3}=0$	A1	Correct quadratic AEF
	$m=2\sqrt{3}$ $m=\frac{7}{\sqrt{3}}$	A1	Correct solutions to the quadratic
	$y=2\sqrt{3}x$ $y=\frac{7}{\sqrt{3}}x$	A1	Correct invariant lines. If M0 then SCB1 for two correct lines.
		5	
1(c)	$\mathbf{M}^{-1}=\frac{1}{14}\begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -7\sqrt{3} & 7\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\frac{1}{14} & \frac{\sqrt{3}}{14}\\ -\sqrt{3} & 1\end{pmatrix}$	M1 A1	SCB1 for finding the inverse of 2**M**
		2	

Question	Answer	Marks	Guidance
3(a)	[One-way] stretch, shear	B1	Both types.
	Stretch followed by shear	B1	Correct order.
	Stretch parallel to the x-axis, scale factor 7	B1	
	Shear, x-axis fixed, with $(0,1)$ mapped to $(2,1)$.	B1	
		4	

Question	Answer	Marks	Guidance
3(b)	$\mathbf{M}=\begin{pmatrix}7 & 2\\ 0 & 1\end{pmatrix}$	B1	
	$\begin{pmatrix}7 & 2\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}7x+2y\\ y\end{pmatrix}$	B1 FT	Transforms $\begin{pmatrix}x\\ y\end{pmatrix}$ to $\begin{pmatrix}X\\ Y\end{pmatrix}$.
	$mx=m(7x+2mx)$	M1	Uses $y=mx$ and $Y=mX$.
	$2m^2+6m=0$	A1	
	$y=0$ and $y=-3x$	A1	SCB1 if M0 and both straight lines correct
		5	
3(c)	Area of PQR = $\lvert 7\rvert\times$ Area of DEF	M1	
	Area of $DEF=5$ cm^2	A1	SCB1 for an answer of 245 (35×7)
		2	

Question	Answer	Marks	Guidance
4(a)	$\sum_{r=1}^{n} e^{rx}\left(e^{2x}-2e^{x}+1\right)=e^{3x}-2e^{2x}+e^{x}$ $+e^{4x}-2e^{3x}+e^{2x}$ $+e^{5x}-2e^{4x}+e^{3x}$ $\vdots$ $+e^{(n+1)x}-2e^{nx}+e^{(n-1)x}$ $+e^{(n+2)x}-2e^{(n+1)x}+e^{nx}$	M1 A1	Shows enough complete terms to make pattern of cancelling clear GP method. $\left(e^{2x}-2e^{x}+1\right)\sum_{r=1}^{n} e^{rx}=\left(e^{2x}-2e^{x}+1\right)e^{x}\frac{(e^{x})^{n}-1}{e^{x}-1}$
	$=e^{x}-e^{2x}-e^{(n+1)x}+e^{(n+2)x}$	A1	OE $e^{x}(e^{x}-1)(e^{nx}-1)$
		3	
4(b)	$x<0$	B1	Accept $x \leqslant 0$.
	$e^{nx}\to 0$ as $n\to\infty$ leading to $u_1+u_2+u_3+\ldots=e^{x}-e^{2x}$	M1 A1	Must see $e^{nx}\to 0\,[\text{as } n\to\infty]$ agreeing with their set of x
		3	
4(c)	$\sum_{r=1}^{n}\ln u_r=\sum_{r=1}^{n}(rx+\ln(e^{x}-1)^{2})$	M1*	Uses laws of logarithms correctly.
	$\sum_{r=1}^{n}\ln u_r=\frac{1}{2}xn(n+1)+n\ln(e^{x}-1)^{2}$	dM1 A1	Applies $\sum_{r=1}^{n} r=\frac{1}{2}n(n+1)$. AEF for $+n\ln(e^{x}-1)^{2}$
		3	