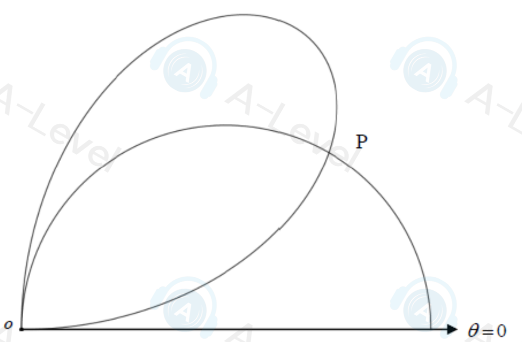
Question	Answer	Marks	Guidance
3(a)	$8r^3+12r^2+6r+1-\left(8r^3-12r^2+6r-1\right)=24r^2+2$	M1	Expands.
	$24\sum_{r=1}^{n} r^2+2n=(2n+1)^3-1$	M1 A1 A1	Sums both sides and uses method of differences with sufficient complete terms.
	$24\sum_{r=1}^{n} r^2=8n^3+12n^2+4n=4n(n+1)(2n+1)$	A1	AG.
		5	

Question	Answer	Marks	Guidance
3(b)	$S_{2n}=\sum_{r=1}^{2n} r^2+2\sum_{r=1}^{n}(2r)^2=\sum_{r=1}^{2n} r^2+8\sum_{r=1}^{n} r^2$	M1	Relates with sum of squares.
	$S_{2n}=\frac{1}{6}(2n)(2n+1)(4n+1)+\frac{8}{6}n(n+1)(2n+1)$	M1	Applies $\sum_{r=1}^{n} r^2=\frac{1}{6}n(n+1)(2n+1)$.
	$S_{2n}=\frac{1}{3}n(2n+1)(4n+1+4n+4)=\frac{1}{3}n(2n+1)(8n+5)$	A1	$\sum_{r=1}^{n} r\ =\frac{1}{2}n(n+1)$.
	Alternative for 3(b)		
	$S_{2n}=\sum_{r=1}^{n}(2r-1)^2+3\sum_{r=1}^{n}(2r)^2=\sum_{r=1}^{n}\left(4r^2-4r+1+12r^2\right)$	M1	Relates with sum of squares.
	$\frac{16n(n+1)(2n+1)}{6}-4\frac{n(n+1)}{2}+n$	M1	Applies $\sum_{r=1}^{n} r^2=\frac{1}{6}n(n+1)(2n+1)$ and $\sum_{r=1}^{n} r\ =\frac{1}{2}n(n+1)$
	$\frac{n}{3}\left(16n^2+18n+5\right)=\frac{n}{3}(2n+1)(8n+5)$	A1	
		3	
3(c)	$\frac{16}{3}$	B1 FT	
		1	

Question	Answer	Marks	Guidance
6(a)	$x^2-x+\frac{1}{4}+y^2=\frac{1}{4}\Rightarrow r^2-r\cos\theta+\frac{1}{4}=\frac{1}{4}$	B1	Uses $x^2+y^2=r^2$ and $x=r\cos\theta$.
	$r(r-\cos\theta)=0$	M1	Factorises.
	$[r\neq 0\Rightarrow]\,r=\cos\theta$	A1	AG.
		3	

Question	Answer	Marks	Guidance
6(b)	$\sin 2\theta=\cos\theta\Rightarrow 2\sin\theta\cos\theta=\cos\theta$	M1	Sets r values equal and uses $\sin 2\theta=2\sin\theta\cos\theta$.
	$\cos\theta\neq 0\Rightarrow\sin\theta=\frac{1}{2}$	A1	$\cos\theta\neq 0$ must be recognised.
	$\left(\frac{1}{2}\sqrt{3},\frac{1}{6}\pi\right)$	A1	
		3	
6(c)		B1	Initial line drawn and one curve correct.
		B1	Other curve correct.
		B1	Intersection marked in correct position and both curves labelled.
		3	

Question	Answer	Marks	Guidance
6(d)	$\frac{1}{2}\int_0^{\frac{1}{6}\pi}\sin^2 2\theta\,\mathrm{d}\theta+\frac{1}{2}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}\cos^2\theta\,\mathrm{d}\theta$	M1	Uses $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ with correct limits.
	$\frac{1}{2}\int_0^{\frac{1}{6}\pi}\sin^2 2\theta\,\mathrm{d}\theta=\frac{1}{4}\int_0^{\frac{1}{6}\pi}1-\cos 4\theta\,\mathrm{d}\theta$	M1	Integrates $\sin^2 2\theta$ using identity.
	$=\frac{1}{4}\left[\theta-\frac{1}{4}\sin 4\theta\right]_0^{\frac{1}{6}\pi}$	A1	
	$\frac{1}{2}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}\cos^2\theta\,\mathrm{d}\theta=\frac{1}{4}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}1+\cos 2\theta\,\mathrm{d}\theta$	M1	Integrates $\cos^2\theta$ using identity.
	$=\frac{1}{4}\left[\theta+\frac{1}{2}\sin 2\theta\right]_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}$	A1	
	$\frac{1}{4}\left(\frac{1}{6}\pi-\frac{1}{8}\sqrt{3}\right)+\frac{1}{4}\left(\frac{1}{2}\pi-\frac{1}{6}\pi-\frac{1}{4}\sqrt{3}\right)=\frac{1}{8}\left(\pi-\frac{3}{4}\sqrt{3}\right)$	A1	
		6	

Question	Answer	Marks	Guidance
4(a)	$\sum_{r=1}^{n}\mathrm{e}^{rx}\left(\mathrm{e}^{2x}-2\mathrm{e}^{x}+1\right)=\mathrm{e}^{3x}-2\mathrm{e}^{2x}+\mathrm{e}^{x}$ $+\mathrm{e}^{4x}-2\mathrm{e}^{3x}+\mathrm{e}^{2x}$ $+\mathrm{e}^{5x}-2\mathrm{e}^{4x}+\mathrm{e}^{3x}$ $\vdots$ $+\mathrm{e}^{(n+1)x}-2\mathrm{e}^{nx}+\mathrm{e}^{(n-1)x}$ $+\mathrm{e}^{(n+2)x}-2\mathrm{e}^{(n+1)x}+\mathrm{e}^{nx}$	M1 A1	Shows enough complete terms to make pattern of cancelling clear GP method. $\left(\mathrm{e}^{2x}-2\mathrm{e}^{x}+1\right)\sum_{r=1}^{n}\mathrm{e}^{rx}=\left(\mathrm{e}^{2x}-2\mathrm{e}^{x}+1\right)\mathrm{e}^{x}\frac{(\mathrm{e}^{x})^{n}-1}{\mathrm{e}^{x}-1}$
	$=\mathrm{e}^{x}-\mathrm{e}^{2x}-\mathrm{e}^{(n+1)x}+\mathrm{e}^{(n+2)x}$	A1	OE $e^{x}(e^{x}-1)(e^{nx}-1)$
		3	
4(b)	$x<0$	B1	Accept $x\leqslant 0$.
	$\mathrm{e}^{nx}\to 0$ as $n\to\infty$ leading to $u_1+u_2+u_3+\ldots=\mathrm{e}^{x}-\mathrm{e}^{2x}$	M1 A1	Must see $\mathrm{e}^{nx}\to 0$ [as $n\to\infty$] agreeing with their set of x
		3	
4(c)	$\sum_{r=1}^{n}\ln u_r=\sum_{r=1}^{n}(rx+\ln(\mathrm{e}^{x}-1)^2)$	M1*	Uses laws of logarithms correctly.
	$\sum_{r=1}^{n}\ln u_r=\frac{1}{2}xn(n+1)+n\ln(\mathrm{e}^{x}-1)^2$	dM1 A1	Applies $\sum_{r=1}^{n}r=\frac{1}{2}n(n+1)$. AEF for $+n\ln(\mathrm{e}^{x}-1)^2$
		3	

Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix}1&3\\2&5\end{vmatrix}-k\begin{vmatrix}2&3\\3&5\end{vmatrix}+3\begin{vmatrix}2&1\\3&2\end{vmatrix}=0\Rightarrow -1-k+3=0\Rightarrow k=2$	M1 A1	Sets determinant of **A** equal to zero.
	$\begin{pmatrix}-2&-1&1\\1&1&3\end{pmatrix}\begin{pmatrix}1&2&3\\2&1&3\\3&2&5\end{pmatrix}\begin{pmatrix}0&-2\\-1&3\\0&0\end{pmatrix}=\begin{pmatrix}-2&-1&1\\1&1&3\end{pmatrix}\begin{pmatrix}-2&4\\-1&-1\\-2&0\end{pmatrix}$	M1	Multiplying two matrices correctly, correct dimensions.
	$\begin{pmatrix}3&-7\\-9&3\end{pmatrix}$	M1 A1	Completing matrix multiplication, AG.
		5	
5(b)	$\begin{pmatrix}3&-7\\-9&3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}3x-7y\\-9x+3y\end{pmatrix}$	B1	Transforms $\begin{pmatrix}x\\y\end{pmatrix}$ to $\begin{pmatrix}X\\Y\end{pmatrix}$.
	$-9x+3mx=m(3x-7mx)$	M1 A1	Uses $y=mx$ and $Y=mX$.
	$-9+3m=3m-7m^2\Rightarrow 7m^2=9$	A1	
	$y=\frac{3}{\sqrt{7}}x$ and $y=-\frac{3}{\sqrt{7}}x$	A1	
		5	

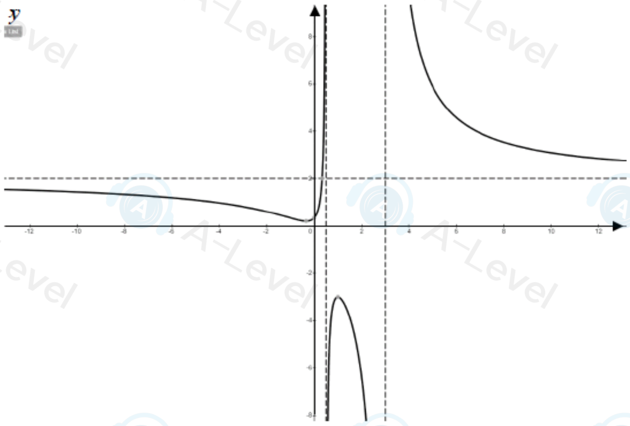
Question	Answer	Marks	Guidance
5(c)	$\mathbf{D}=\begin{pmatrix}\alpha & 0\\ 0 & \alpha\end{pmatrix}$	B1	
	$\mathbf{E}=\begin{pmatrix}\beta & 0\\ 0 & 1\end{pmatrix}$	B1	
	$\mathbf{F}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	B1	
	$\begin{pmatrix}3 & -7\\ -9 & 3\end{pmatrix}=\begin{pmatrix}\alpha & 0\\ 0 & \alpha\end{pmatrix}-9\begin{pmatrix}0 & \beta\\ 1 & 0\end{pmatrix}$	M1	Setting up simultaneous equations using their **D** and **E**.
	$\mathbf{D}=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$ $\mathbf{E}=\begin{pmatrix}\frac{7}{9} & 0\\ 0 & 1\end{pmatrix}$	A1	Condone $\alpha=3, \beta=\frac{7}{9}$ if it is clear that they refer to the correct matrices.
		5	

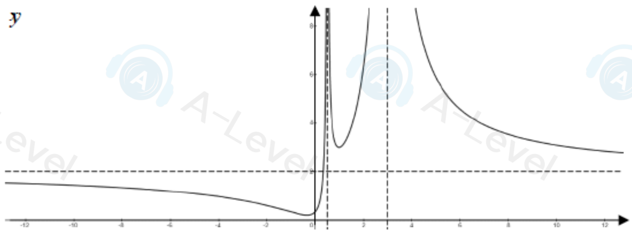
Question	Answer	Marks	Guidance
1(a)	$2\alpha+\gamma=-b$ $\alpha^2+2\alpha\gamma=0$	B1	
	$\alpha=-2\gamma$ leading to $-4\gamma+\gamma=-b$	M1	Solves simultaneous equations, or, express b and d in terms of α and γ.
	$\gamma=\frac{1}{3}b,\ \alpha=-\frac{2}{3}b$	A1	$b=3\gamma=-\dfrac{3}{2}\alpha$ and $d=-\alpha^2\gamma$.
	$\alpha^2\gamma=-d$ leading to $\frac{4}{27}b^3=-d$ leading to $4b^3+27d=0$	M1 A1	Substitutes into third equation, AG.
		5	
1(b)	$3b=b^2$ leading to $b=3$	M1 A1	Uses $2\alpha^2+\gamma^2=(2\alpha+\gamma)^2-2(\alpha^2+2\alpha\gamma)$ or substitutes for α,γ in terms of b.
	$d=-4$	A1	
		3	

2(a)	$S_1=2$	B1	
	$S_2=S_1{}^2-2(0)$	M1	Uses formula for sum of squares.
	$=4$	A1	Correct answer implies M1A1.
		3	
2(b)(i)	$S_{n+3}=2S_{n+2}-\frac{3}{2}S_n$	B1	CAO or as a single fraction.
		1	
2(b)(ii)	$S_4=2S_3-\frac{3}{2}S_1=2(2S_2-\frac{3}{2}S_0)-\frac{3}{2}S_1$	M1	Uses their recursive formula from part **(i)** to find S_4 $\left[S_3=\frac{7}{2}\right]$.
	$=4$	A1	
		2	
2(c)	$x=2-y$	B1	SOI
	$2(2-y)^3-4(2-y)^2+3=0$	M1	Makes *their* substitution.
	$2y^3-8y^2+8y-3=0$	A1	OE but must be an equation.
		3	

Question	Answer	Marks	Guidance
2(d)	$\dfrac{\frac{8}{2}}{\frac{-3}{2}}$ OR Or use $2S_2-8S_1+8S_0-3S_{-1}=0$ with substitution of their values	M1	Uses $\dfrac{1}{\alpha'}+\dfrac{1}{\beta'}+\dfrac{1}{\gamma'}=\dfrac{\alpha'\beta'+\beta'\gamma'+\gamma'\alpha'}{\alpha'\beta'\gamma'}$.
	$=\dfrac{8}{3}$	A1 FT	FT from **2(c)**.
		2	

Question	Answer	Marks	Guidance
6(a)	$x=\frac{1}{2},\ x=3$	B1	Vertical asymptotes.
	$y=2$	B1	Horizontal asymptote.
		2	

Question	Answer	Marks	Guidance
6(b)	$\frac{dy}{dx}=\frac{(2x^2-7x+3)(8x+1)-(4x^2+x+1)(4x-7)}{\left(2x^2-7x+3\right)^2}$	M1	Finds $\frac{dy}{dx}$. Allow top line only for M1.
	$-3x^2+2x+1=0$	M1	Sets equal to 0 and forms equation.
	$\left(-\frac{1}{3},\frac{1}{5}\right),\ (1,-3)$	A1 A1	
		4	
6(c)		B1	Axes and asymptotes. Clear identification (label or clear intersection with axes at correct place).
		B1	$x>3$ correctly approaching asymptotes, not too truncated.
		B1	$\frac{1}{2}<x<3$ correct.
		B1	$x<\frac{1}{2}$ correct.
	$\left(0,\frac{1}{3}\right)$	B1	States coordinates of intersection with axis. May be seen on their graph.
		5	

Question	Answer	Marks	Guidance
6(d)		B1FT	FT from sketch in (c). At least two branches.
	$k>3$	B1	
		2	