Question	Answer	Marks	Guidance
3(a)	$5\times1^4+1^2=\frac{1}{2}(2)^2(2+1)[=6]$ so H_1 is true.	B1	Checks base case.
	Assume that $\sum_{r=1}^{k}\left[\left(5r^4+r^2\right)\right]=\frac{1}{2}k^2(k+1)^2(2k+1)$	B1	States inductive hypothesis [for some k] including the algebraic form. If says for **ALL** k, then B0.
	$\sum_{r=1}^{k+1}\left[\left(5r^4+r^2\right)\right]=\frac{1}{2}k^2(k+1)^2(2k+1)+5(k+1)^4+(k+1)^2$	M1	Considers sum to $k+1$.
	$\frac{1}{2}(k+1)^2\left(2k^3+k^2+10(k+1)^2+2\right)$	M1	Take out the factor of $(k+1)^2$ OR expands the summation expression and the target expression for $k+1$ and collects like terms for both.
	$\frac{1}{2}(k+1)^2(2k^3+11k^2+20k+12)=\frac{1}{2}(k+1)^2(k+2)^2(2k+3)$	A1	Factorises or having expanded, checks explicitly. At least one intermediate step seen following the award of M1 before reaching the answer.
	So H_{k+1} is true. By induction, H_n is true for all positive integers n.	A1	States conclusion. Implication must be clearly expressed.
		6	
3(b)	$5\sum_{r=1}^{n}r^4+\frac{1}{6}n(n+1)(2n+1)=\frac{1}{2}n^2(n+1)^2(2n+1)$	M1	Uses correct formula for $\sum r^2$.
	$[5]\sum_{r=1}^{n}r^4=\frac{1}{6}n(n+1)(2n+1)(3n(n+1)-1)$	M1	Makes $\sum r^4$ the subject and takes out all linear factors and the remaining term is of correct form.
	$\sum_{r=1}^{n}r^4=\frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$	A1	CAO
		3	

Question	Answer	Marks	Guidance
2	$\frac{\mathrm{d}\left(\tan^{-1}x\right)}{\mathrm{d}x}=\frac{1}{1+x^2}$ so true when $n=1$.	B1	Differentiates once.
	Assume that $\frac{\mathrm{d}^k}{\mathrm{d}x^k}\left(\tan^{-1}x\right)=\mathrm{P}_k(x)\left(1+x^2\right)^{-k}$, where $\deg \mathrm{P}_k(x)=k-1$.	B1	States inductive hypothesis. Must have $\deg \mathrm{P}_k(x)=k-1$.
	$\frac{\mathrm{d}^{k+1}\left(\tan^{-1}x\right)}{\mathrm{d}x^{k+1}}=\mathrm{P}_k'(x)\left(1+x^2\right)^{-k}-2kx\mathrm{P}_k(x)\left(1+x^2\right)^{-k-1}$	M1 A1	Differentiates kth derivative using the product rule.
	$=\left(\mathrm{P}_k'(x)\left(1+x^2\right)-2kx\mathrm{P}_k(x)\right)\left(1+x^2\right)^{-k-1}$ so $\deg \mathrm{P}_{k+1}(x)=k$	A1	Writes in the form $\mathrm{P}_{k+1}(x)\left(1+x^2\right)^{-k-1}$.
	So true when $n=k+1$. By induction, true for all positive integers n.	A1	Attempts to show degree of $\mathrm{P}_{k+1}(x)$ is at most k (condone not showing coefficient of x^k is non-zero) and states conclusion.
		6	

Question	Answer	Marks	Guidance
3(a)	$a_2=2^3$ leading to $\ln a_2=3\ln 2$ so true when $n=2$.	B1	Checks base case. Must be $n=2$ and exact.
	Assume that $\ln a_k \geqslant 3^{k-1}\ln 2$.	B1	States inductive hypothesis.
	$\ln a_{k+1}=3\ln\left(a_k+\frac{1}{a_k}\right)>3\ln a_k$	M1	Applies result given in part **(a)**.
	$\geqslant 3^k\ln 2$	M1 A1	Applies inductive hypothesis and writes in required form.
	So true when $n=k+1$. By induction, true for all integers $n \geqslant 2$.	A1	States conclusion, all previous marks must be gained.
		6	
3(b)	$3\ln\left(a_n+\frac{1}{a_n}\right)-\ln a_n>2\ln a_n$	M1	Applies results given in parts **(a)** and **(b)**.
	$>2\times3^{n-1}\ln 2=3^{n-1}\ln 4$	A1	AG
		2	

Question	Answer	Marks	Guidance
2	$1=\frac{1-2x+x^2}{(1-x)^2}=\frac{(1-x)^2}{(1-x)^2}$ so H_1 is true.	B1	Checks base case.
	Assume that $\sum_{r=1}^{k} rx^{r-1}=\frac{1-(k+1)x^k+kx^{k+1}}{(1-x)^2}$.	B1	States inductive hypothesis.
	$\sum_{r=1}^{k+1} rx^{r-1}=\frac{1-(k+1)x^k+kx^{k+1}}{(1-x)^2}+(k+1)x^k$	M1	Considers sum to $k+1$.
	$\frac{1-(k+1)x^k+kx^{k+1}+(k+1)x^k\left(1-2x+x^2\right)}{(1-x)^2}$	M1	Puts over a common denominator.
	$\frac{1+kx^{k+1}+(k+1)x^k\left(-2x+x^2\right)}{(1-x)^2}=\frac{1-(k+2)x^{k+1}+(k+1)x^{k+2}}{(1-x)^2}$	A1	
	So H_{k+1} is true. By induction, H_n is true for all positive integers n.	A1	States conclusion.
		6	

Question	Answer	Marks	Guidance
5	$\frac{d}{dx}(x\sin x)=x\cos x+\sin x=(-1)^0(x\cos x+(2(1)-1)\sin x)$	B1	Checks base case using product rule.
	Assume true for $n=k$, so $\frac{d^{2k-1}}{dx^{2k-1}}(x\sin x)=(-1)^{k-1}(x\cos x+(2k-1)\sin x)$	B1	States inductive hypothesis.
	Then $\frac{d^{2k}}{dx^{2k}}(x\sin x)=(-1)^{k-1}(-x\sin x+2k\cos x)$	M1 A1	Differentiates once. Must have correct LHS for A1.
	$\frac{d^{2k+1}}{dx^{2k+1}}(x\sin x)=(-1)^{k-1}(-x\cos x-\sin x-2k\sin x)$ $=(-1)^k(x\cos x+(2k+1)\sin x)$	M1 A1	Differentiates again.
	So, it is also true for $n=k+1$. Hence, by induction, true for all positive integers.	A1	States conclusion.
		7	