

7 The curve C has equation $y = \frac{5x^2}{5x-2}$.

(a) Find the equations of the asymptotes of C .

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(b) Find the coordinates of the stationary points on C .

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(c) Sketch C .

[3]

(d) Sketch the curve with equation $y = \left| \frac{5x^2}{5x-2} \right|$ and find in exact form the set of values of x for which $\left| \frac{5x^2}{5x-2} \right| < 2$. [6]

7 The position vectors of the points A, B, C, D are

$$7\mathbf{i} + 4\mathbf{j} - \mathbf{k},$$

$$11\mathbf{i} + 3\mathbf{j},$$

$$2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k},$$

$$2\mathbf{i} + 7\mathbf{j} + \lambda\mathbf{k}$$

respectively.

(a) Given that the shortest distance between the line AB and the line CD is 3, show that $\lambda^2 - 5\lambda + 4 = 0$. [7]

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Let Π_1 be the plane ABD when $\lambda = 1$.

Let Π_2 be the plane ABD when $\lambda = 4$.

(b) (i) Write down an equation of Π_1 , giving your answer in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$. [2]

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(ii) Find an equation of Π_2 , giving your answer in the form $ax + by + cz = d$. [4]

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(c) Find the acute angle between Π_1 and Π_2 . [5]

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5 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} + 4\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2 . [5]

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The plane Π_1 contains l_1 and is parallel to l_2 .

(b) Obtain an equation of Π_1 in the form $px + qy + rz = s$. [2]

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(c) The point $(1, 1, 1)$ lies on the plane Π_2 .

It is given that the line of intersection of the planes Π_1 and Π_2 passes through the point $(0, 0, 2)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.

Obtain an equation of Π_2 in the form $ax + by + cz = d$. [3]

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5 The plane Π has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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The point P has position vector $4\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$.

(b) Find the position vector of the foot of the perpendicular from P to Π . [4]

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(c) The line l is parallel to the vector $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

Find the acute angle between l and Π . [3]

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7 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ respectively. The plane Π_1 contains l_1 and is parallel to l_2 .

(a) Find the equation of Π_1 , giving your answer in the form $ax + by + cz = d$. [4]

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The plane Π_2 contains l_2 and the point with coordinates $(2, -1, 7)$.

(b) Find the acute angle between Π_1 and Π_2 . [4]

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The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 .

(c) Find a vector equation for PQ . [7]

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4 The lines l_1 and l_2 have equations

$$\mathbf{r} = -2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k} + \lambda(-4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$$

respectively.

(a) Find the shortest distance between l_1 and l_2 .

[5]

The plane Π contains l_1 and the point with position vector $-\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$.

(b) Find an equation of Π , giving your answer in the form $ax + by + cz = d$.

[4]

5 The plane Π has equation $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j})$.

- (a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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The line l passes through the point P with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the vector \mathbf{k} .

- (b) Find the position vector of the point where l meets Π . [3]

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