7 The curve C has polar equation $r^2 = \sin 2\theta \cos\theta$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the equation of the line of symmetry. [3]

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(b) Find a Cartesian equation for C. [3]

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(c) Find the total area enclosed by C. [4]

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(d) Find the greatest distance of a point on C from the pole. [6]

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5 The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

(b) Find the perpendicular distance from the point with position vector $-\mathbf{i} - 2\mathbf{k}$ to Π_1. [3]

(c) The plane Π_2 has equation $3x + 2y - z = 14$.

Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

7 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ respectively. The plane Π_1 contains l_1 and is parallel to l_2.

(a) Find the equation of Π_1, giving your answer in the form $ax + by + cz = d$. [4]

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The plane Π_2 contains l_2 and the point with coordinates $(2, -1, 7)$.

(b) Find the acute angle between Π_1 and Π_2. [4]

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The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2.

(c) Find a vector equation for PQ. [7]

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5 The curve C has polar equation $r = a\tan\theta$, where a is a positive constant and $0 \leqslant \theta \leqslant \frac{1}{4}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{4}\pi$. [4]

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(c) Show that C has Cartesian equation $y = \dfrac{x^2}{\sqrt{a^2 - x^2}}$. [3]

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(d) Using your answer to part **(b)**, deduce the exact value of $\displaystyle\int_0^{\frac{1}{2}a\sqrt{2}} \frac{x^2}{\sqrt{a^2 - x^2}}\,dx$. [2]

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1 The matrix **M** represents the sequence of two transformations in the x-y plane given by a stretch parallel to the x-axis, scale factor 14, followed by a rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.

(a) Show that $2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$. [4]

(b) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **M**. [5]

The unit square S in the x-y plane is transformed by **M** onto the rectangle P.

(c) Find the matrix which transforms P onto S. [2]

3 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 7 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix **M** represents a sequence of two geometrical transformations in the *x*-*y* plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

(b) Find the equations of the invariant lines, through the origin, of the transformation represented by **M**. [5]

The triangle *DEF* in the *x*-*y* plane is transformed by **M** onto triangle *PQR*.

(c) Given that the area of triangle *PQR* is $35\,\text{cm}^2$, find the area of triangle *DEF*. [2]

4 Let $u_r = e^{rx}(e^{2x} - 2e^x + 1)$.

(a) Using the method of differences, or otherwise, find $\sum_{r=1}^{n} u_r$ in terms of n and x. [3]

(b) Deduce the set of non-zero values of x for which the infinite series

$$u_1 + u_2 + u_3 + \ldots$$

is convergent and give the sum to infinity when this exists. [3]

(c) Using a standard result from the list of formulae (MF19), find $\sum_{r=1}^{n} \ln u_r$ in terms of n and x. [3]