3 **(a)** By considering $(2r+1)^3-(2r-1)^3$, use the method of differences to prove that

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1).$$ [5]

Let $S_n = 1^2+3\times2^2+3^2+3\times4^2+5^2+3\times6^2+\ldots+\left(2+(-1)^n\right)n^2$.

(b) Show that $S_{2n} = \frac{1}{3}n(2n+1)(an+b)$, where a and b are integers to be determined. [3]

(c) State the value of $\lim\limits_{n\to\infty}\frac{S_{2n}}{n^3}$. [1]

6 **(a)** Show that the curve with Cartesian equation

$$\left(x-\tfrac{1}{2}\right)^2+y^2=\tfrac{1}{4}$$

has polar equation $r=\cos\theta$. [3]

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The curves C_1 and C_2 have polar equations

$$r=\cos\theta \quad \text{and} \quad r=\sin 2\theta$$

respectively, where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole and at another point P.

(b) Find the polar coordinates of P. [3]

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(c) In a single diagram sketch C_1 and C_2, clearly identifying each curve, and mark the point P. [3]

(d) The region R is enclosed by C_1 and C_2 and includes the line OP.

Find, in exact form, the area of R. [6]

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4 Let $u_r = e^{rx}(e^{2x} - 2e^{x} + 1)$.

(a) Using the method of differences, or otherwise, find $\sum_{r=1}^{n} u_r$ in terms of n and x. [3]

(b) Deduce the set of non-zero values of x for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

(c) Using a standard result from the list of formulae (MF19), find $\sum_{r=1}^{n} \ln u_r$ in terms of n and x. [3]

5 Let k be a constant. The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & k & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \qquad \text{and} \qquad \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

It is given that **A** is singular.

(a) Show that $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$. [5]

(b) Find the equations of the invariant lines, through the origin, of the transformation in the $x-y$ plane represented by **CAB**. [5]

(c) The matrices **D**, **E** and **F** represent geometrical transformations in the $x-y$ plane.

- **D** represents an enlargement, centre the origin.
- **E** represents a stretch parallel to the x-axis.
- **F** represents a reflection in the line $y = x$.

Given that $\mathbf{CAB} = \mathbf{D} - 9\mathbf{EF}$, find **D**, **E** and **F**. [5]

1 The cubic equation $x^3 + bx^2 + d = 0$ has roots α, β, γ, where $\alpha = \beta$ and $d \neq 0$.

(a) Show that $4b^3 + 27d = 0$. [5]

(b) Given that $2\alpha^2 + \gamma^2 = 3b$, find the values of b and d. [3]

2 The cubic equation $2x^3 - 4x^2 + 3 = 0$ has roots α, β, γ. Let $S_n = \alpha^n + \beta^n + \gamma^n$.

(a) State the value of S_1 and find the value of S_2. [3]

(b) **(i)** Express S_{n+3} in terms of S_{n+2} and S_n. [1]

(ii) Hence, or otherwise, find the value of S_4. [2]

(c) Use the substitution $y = S_1 - x$, where S_1 is the numerical value found in part **(a)**, to find and simplify an equation whose roots are $\alpha+\beta$, $\beta+\gamma$, $\gamma+\alpha$. [3]

(d) Find the value of $\frac{1}{\alpha+\beta}+\frac{1}{\beta+\gamma}+\frac{1}{\gamma+\alpha}$. [2]

6 The curve C has equation $y = \dfrac{4x^2 + x + 1}{2x^2 - 7x + 3}$.

(a) Find the equations of the asymptotes of C. [2]

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(b) Find the coordinates of any stationary points on C. [4]

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(c) Sketch C, stating the coordinates of any intersections with the axes. [5]

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(d) Sketch the curve with equation $y = \left|\frac{4x^2 + x + 1}{2x^2 - 7x + 3}\right|$ and state the set of values of k for which $\left|\frac{4x^2 + x + 1}{2x^2 - 7x + 3}\right| = k$ has 4 distinct real solutions. [2]