

- 3 (a) Prove by mathematical induction that, for all positive integers n ,

$$\sum_{r=1}^n (5r^4 + r^2) = \frac{1}{2}n^2(n+1)^2(2n+1). \quad [6]$$

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- (b) Use the result given in part (a) together with the List of formulae (MF19) to find $\sum_{r=1}^n r^4$ in terms of n , fully factorising your answer. [3]

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- 2 Prove by mathematical induction that, for all positive integers n ,

$$\frac{d^n}{dx^n}(\tan^{-1}x) = P_n(x)(1+x^2)^{-n},$$

where $P_n(x)$ is a polynomial of degree $n-1$. [6]

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- 3 The sequence of real numbers $a_1, a_2, a_3, \dots$ is such that $a_1 = 1$ and

$$a_{n+1} = \left(a_n + \frac{1}{a_n}\right)^3.$$

- (a) Prove by mathematical induction that $\ln a_n \geq 3^{n-1} \ln 2$ for all integers $n \geq 2$. [6]

[You may use the fact that $\ln\left(x + \frac{1}{x}\right) > \ln x$ for $x > 0$.]

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- (b) Show that $\ln a_{n+1} - \ln a_n > 3^{n-1} \ln 4$ for $n \geq 2$. [2]

- 2 Prove by mathematical induction that, for all positive integers n ,

$$1 + 2x + 3x^2 + \dots + nx^{n-1} = \frac{1 - (n+1)x^n + nx^{n+1}}{(1-x)^2}. \quad [6]$$

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- 5 Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \sin x) = (-1)^{n-1}(x \cos x + (2n-1)\sin x).$$

[7]
