Question	Answer	Marks	Guidance
1(a)	$f'(x) = -2xe^{-x^2}$	B1	Finds first derivative.
	$f''(x) = 4x^2e^{-x^2} - 2e^{-x^2}$	B1	Finds second derivative.
	$f(0) = 1 \quad f'(0) = 0 \quad f''(0) = -2$	M1	Evaluates derivatives at zero.
	$e^{-x^2} = 1 - x^2$	M1 A1	
		5	
1(b)	$\int_0^{\frac{1}{5}} 1 - x^2 \, dx = \left[x - \frac{1}{3}x^3\right]_0^{\frac{1}{5}} = \frac{74}{375}$	M1 A1	Substitutes $1 - x^2$ or better.
		2	

Question	Answer	Partial Marks	Guidance
7(a)	$I_{-1} = \int_0^{\frac{4}{3}} \left(1 + x^2\right)^{-\frac{1}{2}} dx = \left[\sinh^{-1} x\right]_0^{\frac{4}{3}} = \ln 3$	M1 A1	Recognises integral.
		2	
7(b)	$\frac{d}{dx}\left(x\left(1 + x^2\right)^{\frac{1}{2}n}\right) = nx^2\left(1 + x^2\right)^{\frac{1}{2}n-1} + \left(1 + x^2\right)^{\frac{1}{2}n}$	M1 A1	Uses the product rule to differentiate.
	$n\left(1 + x^2 - 1\right)\left(1 + x^2\right)^{\frac{1}{2}n-1} + \left(1 + x^2\right)^{\frac{1}{2}n}$	M1*	Uses $x^2 = 1 + x^2 - 1$.
	$\left[x\left(1 + x^2\right)^{\frac{1}{2}n}\right]_0^{\frac{4}{3}} = nI_n - nI_{n-2} + I_n$	DM1	Integrates both sides using the limits given. Requires previous method mark.
	$\frac{4}{3}\left(\frac{5}{3}\right)^n = (n+1)I_n - nI_{n-2} \Rightarrow (n+1)I_n = nI_{n-2} + \frac{4}{3}\left(\frac{5}{3}\right)^n$	A1	Substitutes limits and rearranges. AG.
		5	
7(c)	$s = \int_0^{\frac{2}{3}} \sqrt{1 + 4x^2} \, dx$	M1	Forms correct integral with correct limits.
	$\frac{1}{2}\int_0^{\frac{4}{3}} \sqrt{1 + u^2} \, du = \frac{1}{2}I_1$	A1	AG.
	$2I_1 = I_{-1} + \frac{4}{3}\left(\frac{5}{3}\right)$	M1	Applies reduction formula with $n = 1$.
	$s = \frac{1}{4}\ln 3 + \frac{5}{9}$	A1	
		4	

Question	Answer	Marks	Guidance
1	$-x^2 + 4x - 1 = 3 - (x-2)^2$	B1	Completes the square.
	$\int_2^{\frac{7}{2}} \frac{1}{\sqrt{3 - (x-2)^2}} dx = \left[\sin^{-1}\left(\frac{x-2}{\sqrt{3}}\right)\right]_2^{\frac{7}{2}}$	M1 A1	Applies formula. $\left[\sin^{-1}\left(\frac{x-2}{\sqrt{3}}\right)\right]_2^{\frac{7}{2}} = \left[-\cos^{-1}\left(\frac{x-2}{\sqrt{3}}\right)\right]_2^{\frac{7}{2}}$
	$\sin^{-1}\left(\frac{1}{2}\sqrt{3}\right) - \sin^{-1}(0) = \frac{1}{3}\pi$	M1 A1	Uses limits. Answer must be in radians.
		5	

Question	Answer	Partial Marks	Guidance
6(a)	$\int_0^1 (1-x)^2\,\mathrm{d}x < \left(\frac{1}{n}\right)\left(1-\frac{0}{n}\right)^2 + \left(\frac{1}{n}\right)\left(1-\frac{1}{n}\right)^2 + \cdots + \left(\frac{1}{n}\right)\left(1-\frac{n-1}{n}\right)^2$	M1 A1	Forms the sum of the areas of the rectangles. Must have first two terms and last term for A1. Accept $\left(\frac{1}{n}\right)\left(1-\frac{n}{n}\right)^2$ for the last term. Accept written in summation form with correct limits.
	$\frac{1}{n} + \frac{1}{n^3}\sum_{r=1}^{n-1}(n-r)^2 = \frac{1}{n} + \frac{1}{n^3}\sum_{r=1}^{n-1}\left(n^2 - 2nr + r^2\right)$ $\left(= 1 - \frac{2}{n^2}\sum_{r=1}^{n-1} r + \frac{1}{n^3}\sum_{r=1}^{n-1} r^2\right)$	M1 A1	Setting up correct series so that formulae from MF19 can be applied. Can also recognise sum as $\frac{1}{n^3}\sum_{r=1}^{n} r^2$
	$\frac{1}{n} + \frac{n^2(n-1)}{n^3} - \frac{n^2(n-1)}{n^3} + \frac{n(n-1)(2n-1)}{6n^3} = \frac{1}{n} + \frac{2n^2-3n+1}{6n^2} = \frac{2n^2+3n+1}{6n^2}$	A1	AG. Must have gained previous accuracy mark.
		5	

Question	Answer	Partial Marks	Guidance
6(b)	$\int_0^1 (1-x)^2\,\mathrm{d}x > \left(\frac{1}{n}\right)\left(1-\frac{1}{n}\right)^2 + \left(\frac{1}{n}\right)\left(1-\frac{2}{n}\right)^2 + \cdots + \left(\frac{1}{n}\right)\left(1-\frac{n-1}{n}\right)^2$	M1 A1	Forms the sum of the areas of appropriate rectangles. Must have first two and last terms for A1. Accept $\left(\frac{1}{n}\right)\left(1-\frac{n}{n}\right)^2$ for the last term. Accept written in summation form with correct limits.
	$\frac{1}{n^3}\sum_{r=1}^{n-1}\left(n^2 - 2nr + r^2\right) = \frac{n^2(n-1)}{n^3} - \frac{n^2(n-1)}{n^3} + \frac{n(n-1)(2n-1)}{6n^3}$ $\left(= 1 - \frac{2}{n^2}\sum_{r=1}^{n} r + \frac{1}{n^3}\sum_{r=1}^{n} r^2\right)$	M1	Setting up correct series so that formulae from MF19 can be applied. Recognising sum as $\frac{1}{n^3}\sum_{r=1}^{n-1} r^2 = \frac{1}{n^3}\sum_{r=1}^{n} r^2 - \frac{1}{n}$ without wrong working scores M1 A1 M1.
	$\frac{2n^2-3n+1}{6n^2}$	A1	
		4	
6(c)	$U_n - L_n = \frac{1}{n}$	M1	Simplifies $U_n - L_n$ to $\frac{c}{n}$.
	$\frac{1}{n} \to 0$ as $n \to \infty$	A1	AG.
		2	

Question	Answer	Marks	Guidance
3(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x\left(1+x^2\right)^n\right) = 2nx^2\left(1+x^2\right)^{n-1} + \left(1+x^2\right)^n$	M1 A1	Uses the product rule to differentiate.
	$= 2n\left(1+x^2-1\right)\left(1+x^2\right)^{n-1} + \left(1+x^2\right)^n$	*M1	Uses $x^2 = x^2 + 1 - 1$.
	$\left[x\left(1+x^2\right)^n\right]_0^1 = 2nI_n - 2nI_{n-1} + I_n$	DM1	Integrates both sides using the limits given.
	$2^n = (2n+1)I_n - 2nI_{n-1} \Rightarrow (2n+1)I_n = 2^n + 2nI_{n-1}$	A1	Substitutes limits and rearranges. AG.
		5	
3(b)	$I_{-1} = \left[\tan^{-1} x\right]_0^1 = \frac{1}{4}\pi$	B1	
	$-I_{-1} = \frac{1}{2} - 2I_{-2}$	M1 A1	Applies reduction formula with $n = -1$.
	$-\frac{1}{4}\pi = \frac{1}{2} - 2I_{-2} \Rightarrow I_{-2} = \frac{1}{4} + \frac{1}{8}\pi$	A1	
		4	

Question	Answer	Marks	Guidance
2(a)	$\dot{x} = \sinh t \quad \dot{y} = \cosh t$	B1	Differentiates x and y with respect to t.
	$\dot{x}^2 + \dot{y}^2 = \sinh^2 t + \cosh^2 t = \cosh 2t$	M1 A1	Applies $\cosh^2 t + \sinh^2 t = \cosh 2t$. May use exponential form or $\left(1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)^{\frac{1}{2}} \mathrm{d}x = \left(1+\frac{\cosh^2 t}{\sinh^2 t}\right)^{\frac{1}{2}} \sinh t\, \mathrm{d}t$.
	$s = \int_0^{\frac{3}{5}} \sqrt{\cosh 2t}\, \mathrm{d}t$	A1	AG
		4	

Question	Answer	Marks	Guidance
2(b)	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\sqrt{\cosh 2t}\right) = \sinh 2t\sqrt{\operatorname{sech} 2t} = \sinh 2t(\cosh 2t)^{-\frac{1}{2}}$	B1	Finds first derivative.
	$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\sqrt{\cosh 2t}\right) = 2\sqrt{\cosh 2t} - \tanh^2 2t\sqrt{\cosh 2t} = 2\sqrt{\cosh 2t} - \sinh^2 2t\left(\cosh 2t\right)^{-\frac{3}{2}}$	B1	Finds second derivative
	$s = \int_0^{\frac{3}{5}} 1 + t^2\, \mathrm{d}t = \left[t + \frac{1}{3}t^3\right]_0^{\frac{3}{5}}$	M1 A1	Applies Maclaurin's series.
	$= \frac{84}{125} = 0.672$	A1	CWO.
		5	

Question	Answer	Marks	Guidance
4(a)	$\sum_{r=1}^{N} \frac{\ln r}{r^2} = \frac{\ln 2}{4} + \sum_{r=3}^{N} \frac{\ln r}{r^2}$	M1 A1	Compares with sum of the areas of the rectangles. M1 for writing out sum, A1 for considering $\sum_{r=3}^{N} \frac{\ln r}{r^2}$.
	$< \frac{\ln 2}{4} + \int_2^N \frac{\ln x}{x^2} \mathrm{d}x$	M1	Compares with integral.
	$\int_2^N \frac{\ln x}{x^2} \mathrm{d}x = \left[-\frac{\ln x + 1}{x}\right]_2^N$	M1 A1	Finds integral.
	$\sum_{r=1}^{N} \frac{\ln r}{r^2} < \frac{\ln 2}{4} + \left(-\frac{\ln N + 1}{N} + \frac{\ln 2 + 1}{2}\right) = \frac{2 + 3\ln 2}{4} - \frac{1 + \ln N}{N}$	M1 A1	Inserts limits, AG. A1 requires second M1.
		7	
4(b)	$\sum_{r=1}^{N} \frac{\ln r}{r^2} = \sum_{r=2}^{N-1} \frac{\ln r}{r^2} + \frac{\ln N}{N^2} > \int_2^N \frac{\ln x}{x^2} \mathrm{d}x + \frac{\ln N}{N^2}$	M1 A1	Compares with integral. Lower limit of 1 scores M0. Accept $\sum_{r=1}^{N} \frac{\ln r}{r^2} > \int_2^{N+1} \frac{\ln x}{x^2}$.
	$= \frac{\ln 2 + 1}{2} - \frac{\ln N + 1}{N} + \frac{\ln N}{N^2}$	A1	Accept $\frac{\ln 2 + 1}{2} - \frac{\ln(N+1) + 1}{N+1}$.
		3	