1 **(a)** By differentiating e^{-x^2}, find the Maclaurin's series for e^{-x^2} up to and including the term in x^2. [5]

(b) Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2}\,dx$, giving your answer as a rational fraction in its lowest terms. [2]

7 The integral I_n, where n is an integer, is defined by $I_n = \int_0^{\frac{4}{3}} (1+x^2)^{\frac{1}{2}n} \mathrm{d}x$.

(a) Find the exact value of I_{-1} giving your answer in the form $\ln a$, where a is an integer to be determined. [2]

(b) By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left(x(1+x^2)^{\frac{1}{2}n}\right)$, or otherwise, show that

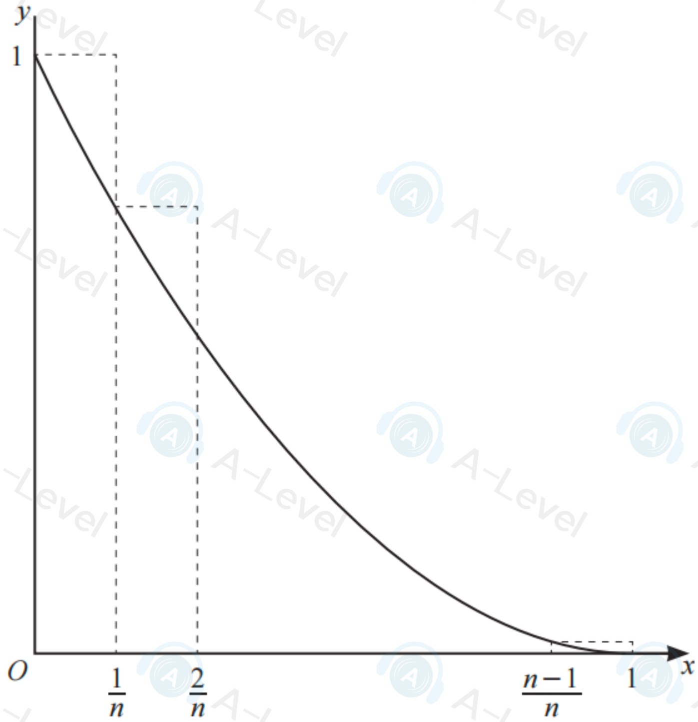
$$(n+1)I_n = nI_{n-2} + \tfrac{4}{3}\left(\tfrac{5}{3}\right)^n.$$ [5]

(c) A curve has equation $y = x^2$, for $0 \leqslant x \leqslant \frac{2}{3}$. The arc length of the curve is denoted by s.

Use the substitution $u = 2x$ to show that $s = \frac{1}{2}I_1$ and find the exact value of s. [4]

1 Find the exact value of $\int_2^{\frac{7}{2}} \frac{1}{\sqrt{4x - x^2 - 1}} \mathrm{d}x$. [5]

6



The diagram shows the curve with equation $y=(1-x)^2$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (1-x)^2 \mathrm{d}x < U_n$, where

$$U_n = \frac{2n^2+3n+1}{6n^2}.$$ [5]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 (1-x)^2 \mathrm{d}x$. [4]

(c) Show that $\lim_{n\to\infty}(U_n - L_n) = 0$. [2]

3 The integral I_n is defined by $I_n = \int_0^1 \left(1+x^2\right)^n \mathrm{d}x$.

(a) By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left(x(1+x^2)^n\right)$, or otherwise, show that

$$(2n+1)I_n = 2^n + 2nI_{n-1}.$$ [5]

(b) Find the exact value of I_{-2}. [4]

2 The curve C has parametric equations

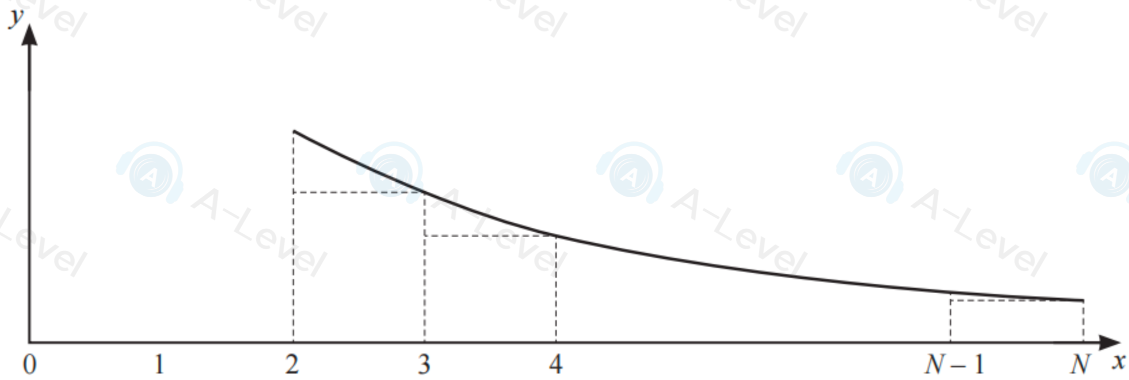
$$x = \cosh t, \quad y = \sinh t, \quad \text{for } 0 < t \leqslant \tfrac{3}{5}.$$

The length of C is denoted by s.

(a) Show that $s = \int_0^{\frac{3}{5}} \sqrt{\cosh 2t}\, dt$. [4]

(b) By finding the Maclaurin's series for $\sqrt{\cosh 2t}$ up to and including the term in t^2, deduce an approximation to s. [5]

4



The diagram shows the curve with equation $y = \frac{\ln x}{x^2}$ for $x \geqslant 2$, together with a set of $(N-2)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{N} \frac{\ln r}{r^2} < \frac{2+3\ln 2}{4} - \frac{1+\ln N}{N}.$$ [7]

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(b) Use a similar method to find, in terms of N, a lower bound for $\sum_{r=1}^{N} \frac{\ln r}{r^2}$. [3]

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