2 Let $I_n = \int_0^1 (1+3x)^n e^{-3x} dx$, where n is an integer.

(a) Show that $3I_n = 1 - 4^n e^{-3} + 3nI_{n-1}$. [3]

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(b) Find the exact value of I_2. [3]

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1 **(a)** By differentiating e^{-x^2}, find the Maclaurin's series for e^{-x^2} up to and including the term in x^2. [5]

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(b) Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2} dx$, giving your answer as a rational fraction in its lowest terms. [2]

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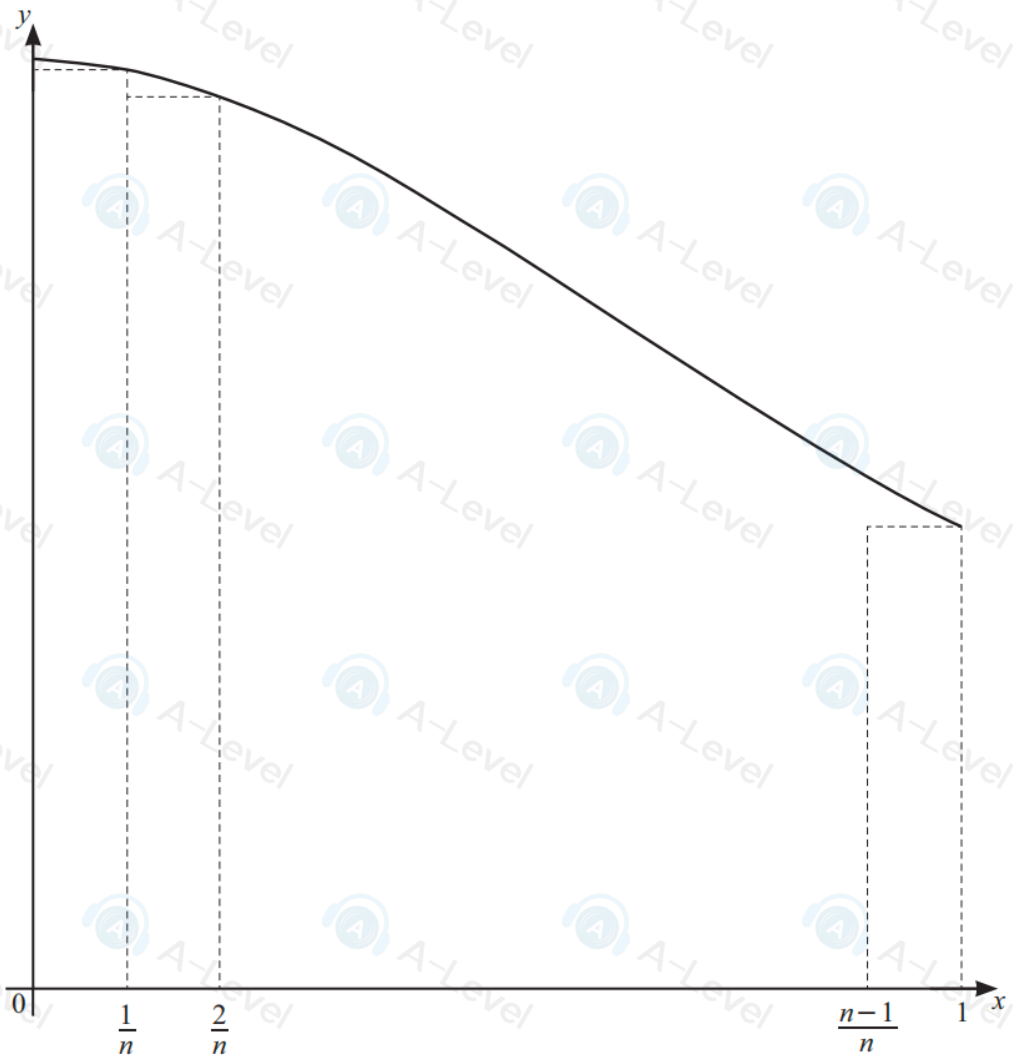

The diagram shows the curve with equation $y = \frac{1}{x^2+1}$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{n}{n^2+r^2} < \frac{1}{4}\pi. \qquad [5]$$

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(b) Use a similar method to find a lower bound for $\sum_{r=1}^{n} \frac{n}{n^2+r^2}$. Give your answer in terms of n and π. [4]

(c) Deduce the exact value of $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2+r^2}$. [1]

3 The integral I_n is defined by $I_n = \int_0^1 \left(1+x^2\right)^n \mathrm{d}x$.

(a) By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left(x(1+x^2)^n\right)$, or otherwise, show that

$$(2n+1)I_n = 2^n + 2nI_{n-1}.$$ [5]

(b) Find the exact value of I_{-2}. [4]

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3 A curve has equation $y = e^x$ for $\ln\frac{4}{3} \leqslant x \leqslant \ln\frac{12}{5}$. The area of the surface generated when the curve is rotated through 2π radians about the x-axis is denoted by A.

(a) Use the substitution $u = e^x$ to show that

$$A = 2\pi \int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1+u^2}\,\mathrm{d}u.$$ [2]

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(b) Use the substitution $u = \sinh v$ to show that

$$A = \pi\left(\frac{904}{225} + \ln\frac{5}{3}\right).$$ [6]

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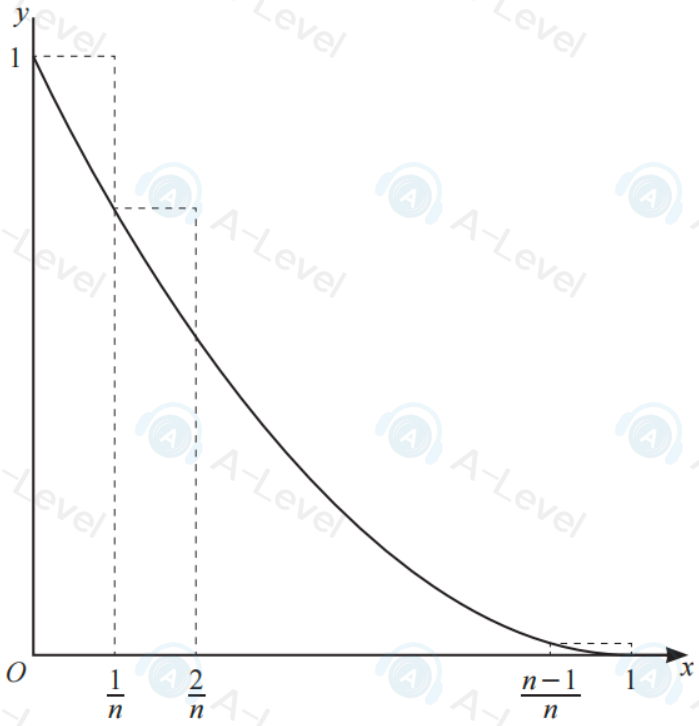

The diagram shows the curve with equation $y = (1-x)^2$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (1-x)^2 \mathrm{d}x < U_n$, where

$$U_n = \frac{2n^2 + 3n + 1}{6n^2}.$$ [5]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 (1-x)^2 \mathrm{d}x$. [4]

(c) Show that $\lim_{n\to\infty}(U_n - L_n) = 0$. [2]

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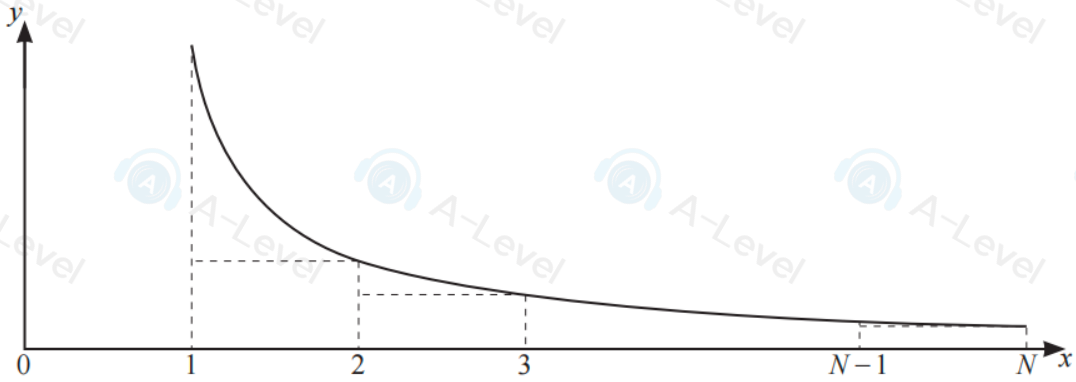

The diagram shows the curve $y = \dfrac{x}{2x^2 - 1}$ for $x \geqslant 1$, together with a set of $N - 1$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{N} \frac{r}{2r^2 - 1} < \tfrac{1}{4}\ln(2N^2 - 1) + 1. \qquad [7]$$

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(b) Use a similar method to find, in terms of N, a lower bound for $\displaystyle\sum_{r=1}^{N} \frac{r}{2r^2 - 1}$. [3]

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