Question	Answer	Marks	Guidance
5(a)	$G_X(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$	**B1**	Accept $(0.5+0.5t)^2$
		1	
5(b)	$P(0H) = \frac{12}{36}$ $P(1H) = \frac{16}{36}$ $P(2H) = \frac{7}{36}$ $P(3H) = \frac{1}{36}$	**M1 A1**	Attempt at probs, at least 2 correct All correct
	$G_Y(t) = \frac{12}{36} + \frac{16}{36}t + \frac{7}{36}t^2 + \frac{1}{36}t^3$	**B1 FT**	FT *their* probabilities, must be cubic with 4 non-zero terms
		3	
5(c)	$G_Z(t) = (\frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2)\left(\frac{12}{36} + \frac{16}{36}t + \frac{7}{36}t^2 + \frac{1}{36}t^3\right)$	**M1**	Attempt to multiply their two PGF
	$= \frac{1}{144}\left(12 + 40t + 51t^2 + 31t^3 + 9t^4 + t^5\right)$	**M1 A1**	Obtain quintic expression and collect terms
		3	
5(d)	$G'_Z(t) = \frac{1}{144}\left(40 + 102t + 93t^2 + 36t^3 + 5t^4\right)$	**M1**	Differentiate
	$E(Z) = G'_Z(1) = \frac{23}{12}$ $(= 1.92)$	**A1**	
		2	
5(e)	2	**B1 FT**	FT power of term with largest coefficient in *their* $G_Z(t)$
		1	

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Question	Answer	Marks	Guidance
4(a)	$G'(t) = \dfrac{4t}{\left(3-2t^2\right)^2}$	**M1**	Differentiate to obtain $\dfrac{kt}{\left(3-2t^2\right)^2}$ OE.
	so $E(X) = G'(1) = 4$	**A1**	CAO WWW
	$G''(t) = \dfrac{12+24t^2}{\left(3-2t^2\right)^3}$ or $4\left(3-2t^2\right)^{-2} + 32t^2\left(3-2t^2\right)^{-3}$	**M1**	OE. Differentiate, allow only numerical or sign slips.
	$\text{Var}(X) = G''(1) + G'(1) - \left(G'(1)\right)^2 = 36 + 4 - 16$	**M1**	Substitute their values into correct formula, dependent on attempt at $G''(t)$.
	$[\text{Var}(X)] = 24$	**A1**	CAO WWW
		5	
4(b)	$G_X(t) = \dfrac{1}{3-2t^2} = \left(3-2t^2\right)^{-1}$ $= \dfrac{1}{3}\left(1+\dfrac{2}{3}t^2+\dfrac{4}{9}t^4+\ldots\right)$	**M1**	Expand given expression or give expression for the term in t^4.
	$P(X=4) =$ their coefficient of t^4	**M1**	Found from legitimate method.
	$[P(X=4) =] \dfrac{4}{27}$	**A1**	WWW
		3	

Question	Answer	Marks	Guidance
5(a)	$P(0\text{ heads}) = \dfrac{8}{30}$, $P(1\text{ head}) = \dfrac{14}{30}$, $P(2\text{ heads}) = \dfrac{7}{30}$, $P(3\text{ heads}) = \dfrac{1}{30}$	**B1**	All correct.
	$G_X(t) = \dfrac{8}{30} + \dfrac{14}{30}t + \dfrac{7}{30}t^2 + \dfrac{1}{30}t^3$	**M1**	Cubic polynomial.
		A1 FT	FT their probabilities that sum to one.
		3	

Question	Answer	Marks	Guidance
5(b)	$G_Y(t) = \frac{25}{36} + \frac{10}{36}t + \frac{1}{36}t^2$	**B1**	
	$G_Z(t) = \frac{1}{30\times36}\left(\frac{8}{30} + \frac{14}{30}t + \frac{7}{30}t^2 + \frac{1}{30}t^3\right)\left(25 + 10t + t^2\right)$	**M1**	With attempt to multiply.
	$\frac{1}{1080}\left(200 + 430t + 323t^2 + 109t^3 + 17t^4 + t^5\right)$	**M1 A1**	Obtain quintic polynomial.
		4	
5(c)	$G_Z'(1) = \frac{1}{1080}(430 + 646 + 327 + 68 + 5)$	**M1**	Differentiate and substitute $t = 1$.
	$\frac{1476}{1080} = \frac{41}{30} = 1.37$	**A1**	
		2	

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Question	Answer	Marks	Guidance
5(a)	$\left[G_X(t) = k\left(1 + 3t + 4t^2\right)\right] \quad k(1+3+4) = 1,\ k = \frac{1}{8}$	**B1**	Working required.
	$G'_X(t) = k(3 + 8t)$,	**M1**	Or $\sum px = 0\times\frac{1}{8} + 1\times\frac{3}{8} + 2\times\frac{4}{8}\left[= \frac{11}{8}\right]$
	$E(X) = G'_X(1) = \frac{11}{8}$	**A1**	AG Evidence of using $t = 1$ or $\sum px$ required. CWO
		3	
5(b)	$G_Z(t) = \frac{1}{8}\left(1 + 3t + 4t^2\right)\times\frac{1}{3}t^2(1 + 2t)$	**M1**	Multiply the two PGFs to obtain a single polynomial of degree 5.
	$\frac{1}{24}\left(t^2 + 5t^3 + 10t^4 + 8t^5\right)$	**A1**	May have t^2 as a factor.
		2	

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Question	Answer	Marks	Guidance
5(c)	$G'_Z(t) = \frac{1}{24}\left(2t + 15t^2 + 40t^3 + 40t^4\right)$ $G''_Z(t) = \frac{1}{24}\left(2 + 30t + 120t^2 + 160t^3\right)$	**M1**	Differentiate twice, allow one slip.
	$\text{Var}(X) = G''_Z(1) + G'_Z(1) - (G'_Z(1))^2 = \frac{1}{24}(312) + \frac{97}{24} - \left(\frac{97}{24}\right)^2$	**M1**	Use correct formula.
	0.707	**A1**	$\frac{407}{576}$
		3	
5(d)	$(Z =)4$	**B1 FT**	FT *their* final polynomial in part **(b)**
		1	

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Question	Answer	Marks	Guidance
6(a)	Probabilities: $\frac{1}{15}$, $\frac{7}{15}$, $\frac{7}{15}$ for 0, 1, 2 heads.	**B1**	
	$G_X(t) = \frac{1}{15} + \frac{7}{15}t + \frac{7}{15}t^2$	**B1 FT**	FT *their* probabilities provided $\sum p = 1$.
	Probabilities: $\frac{8}{15}$, $\frac{7}{15}$ for 1, 2 colours.	**B1**	SC B1 for $\frac{8}{15}$, $\frac{7}{15}$ for 0,1 colours or 0,2 colours.
	$G_Y(t) = \frac{8}{15}t + \frac{7}{15}t^2$	**B1 FT**	FT *their* probabilities provided $\sum p = 1$. SC B1 for $G_Y(t) = \frac{8}{15} + \frac{7}{15}t$ from 0,1 colours or $G_Y(t) = \frac{8}{15} + \frac{7}{15}t^2$ from 0,2 colours.
		4	
6(b)	X and Y are not independent.	**B1**	
		1	

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Question	Answer	Marks	Guidance
6(c)	x | 0 | 1 | 2 y | 1 | 2 | 1 $z=x+y$ | 1 | 3 | 3 $P(Z=z)$ | $\frac{1}{15}$ | $\frac{7}{15}$ | $\frac{7}{15}$	**B1**	Correct 3 cases identified correctly where probabilities are not zero from 1,2 colours. $(x,y)=(0,1)$, $(1,2)$ or $(2,1)$. Award SC B1 for either of the following: $(x,y)=(0,0)$, $(1,1)$ or $(2,0)$ from 0,1 colours. $(x,y)=(0,0)$, $(1,2)$ or $(2,0)$ from 0,2 colours.
		M1	At least one non-zero probability correct. Allow for $\frac{1}{15}$, $\frac{7}{15}$, or $\frac{14}{15}$.
		A1	All probabilities correct.
	$G_Z(t)=\frac{1}{15}t+\frac{14}{15}t^3$	**A1 FT**	FT *their* probabilities provided $\sum p=1$. Allow A1 FT for either of the following: $G_Z(t)=\frac{1}{15}+\frac{14}{15}t^2$ from 0,1 colours. $G_Z(t)=\frac{1}{15}+\frac{7}{15}t^2+\frac{7}{15}t^3$ from 0, 2 colours.
6(d)	$G_Z'(t)=\frac{1}{15}+\frac{42}{15}t^2$ $G_Z'(1)=\frac{1}{15}+\frac{42}{15}$	**M1**	Attempt to differentiate $G_Z(t)$ and find $G_Z'(1)$. FT *their* $G_Z(t)$.
	$\frac{43}{15}$ $[=2.87]$	**A1**	Allow A1 FT for either of the following: $\frac{28}{15}$ $[=1.87]$ from 0,1 colours. $\frac{35}{15}$ $[=2.33]$ from 0,2 colours.
		2	

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Question	Answer	Marks	Guidance
4(a)	$P(0\text{ H}) = \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} = \frac{1}{27}$ $P(1\text{ H}) = \frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}\times 3 = \frac{6}{27}$ $P(2\text{ H}) = \frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}\times 3 = \frac{12}{27}$ $P(3\text{ H}) = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{8}{27}$	M1	At least 2 probabilities correct.
	$G_X(t) = \frac{1}{27} + \frac{6}{27}t + \frac{12}{27}t^2 + \frac{8}{27}t^3$	M1A1	Correct form, ft their probabilities.
	OR: PGF for one coin is $\frac{1}{3} + \frac{2}{3}t$	M1	
	For 3 coins, PGF is $\left(\frac{1}{3} + \frac{2}{3}t\right)^3$	M1A1	
		3	
4(b)	$G_Z(t) = \frac{1}{108}\left(1+2t+t^2\right)\left(1+6t+12t^2+8t^3\right)$	M1	Correct method.
	$\frac{1}{108}\left(1+8t+25t^2+38t^3+28t^4+8t^5\right)$	M1	Obtain quintic polynomial.
	$\frac{1}{108} + \frac{2}{27}t + \frac{25}{108}t^2 + \frac{19}{54}t^3 + \frac{7}{27}t^4 + \frac{2}{27}t^5$	A1	
		3	

Question	Answer	Marks	Guidance
4(c)	$G_Z'(t) = \frac{1}{108}\left(8+50t+114t^2+112t^3+40t^4\right)$ $E(Z) = \frac{1}{108}\left(8+50+114+112+40\right)$	M1	Differentiate and evaluate at $t = 1$
	$\frac{324}{108} = 3$	A1	
	Alternative method for questions 4(c)		
	$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{1}{2} + \frac{1}{2}$	M1	Summing expected values for each coin
	3	A1	
		2	