6 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{8} & 0 \leqslant x < 1, \\ \frac{1}{28}(8-x) & 1 \leqslant x \leqslant 8, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

(b) Find the value of the constant a such that $P(X \leqslant a) = \frac{5}{7}$. [3]

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The random variable Y is given by $Y = \sqrt[3]{X}$.

(c) Find the probability density function of Y. [5]

5 The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ 1 - \frac{1}{144}(12-x)^2 & 0 \leqslant x \leqslant 12, \\ 1 & x > 12. \end{cases}$$

(a) Find the upper quartile of X. [2]

(b) Find $\mathrm{Var}(X^2)$. [5]

The random variable Y is given by $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [3]

5 A continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{16}\sqrt{x} & 0 \leqslant x < 4, \\ \frac{1}{k\sqrt{x}} & 4 \leqslant x \leqslant 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = 3$. [2]

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(b) Find the median value of X. [3]

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The random variable Y is defined by $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [5]

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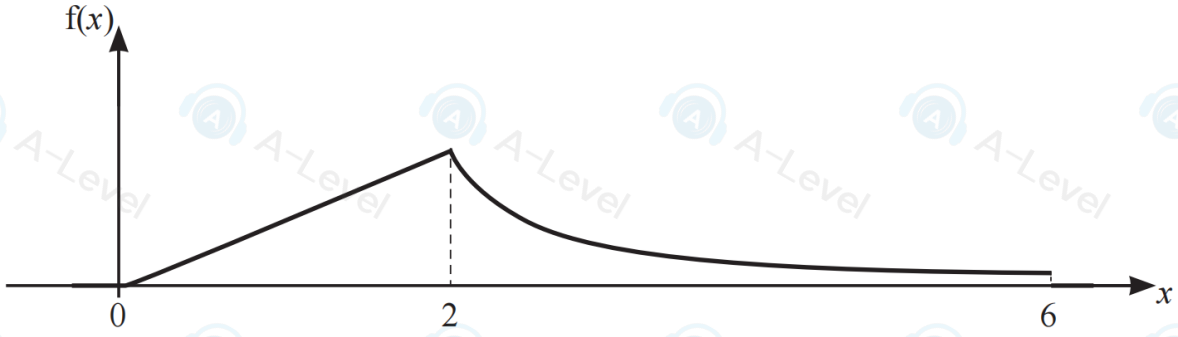

As shown in the diagram, the continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} mx & 0 \leqslant x \leqslant 2, \\ \dfrac{k}{x^2} + c & 2 \leqslant x \leqslant 6, \\ 0 & \text{otherwise}, \end{cases}$$

where m, k and c are constants.

(a) Given that $\mathrm{P}(X \leqslant 2) = \frac{1}{3}$, show that $m = \frac{1}{6}$ and find the values of k and c. [4]

(b) Find the exact numerical value of the interquartile range of X. [5]

5 The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2, \\ \dfrac{(x-2)^2}{12} & 2 \leqslant x < 4, \\ 1 - \dfrac{(8-x)^2}{24} & 4 \leqslant x \leqslant 8, \\ 1 & x > 8. \end{cases}$$

(a) Sketch the graph of the probability density function of X. [3]

(b) Find $\mathrm{E}(X)$. [3]

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(c) Find the exact value of the interquartile range of X. [4]

1 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{6}\left(x^{-\frac{1}{3}} - x^{-\frac{2}{3}}\right) & 1 \leqslant x \leqslant 27, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

The random variable Y is defined by $Y = X^{\frac{1}{3}}$.

(b) Find the probability density function of Y. [3]

(c) Find the exact value of the median of Y. [2]