Question	Answer	Marks	Guidance
10(a)	$x^2+(2x-1)^2-2[=0] \to 5x^2-4x-1\ [=0]$	*M1 A1	Or $5y^2+2y-7\ [=0]$.
	$(5x+1)(x-1)[=0]$ or $(5y+7)(y-1)[=0]$	DM1	May see factors or formula or completing square.
	$x=1,\ y=1$ or (1, 1) only	A1	May be implied on the diagram.
		4	

Question	Answer	Marks	Guidance
10(b)	$(\pi)\int(2-x^2)\mathrm{d}x\ =\ (\pi)\left(2x-\frac{x^3}{3}\right)$	*M1 A1	Attempt integration of y^2, allow $\int(2-y^2)\mathrm{d}y$.
	$(\pi)\left(\left(2\sqrt{2}-\frac{(\sqrt{2})^3}{3}\right)-\left(2-\frac{1}{3}\right)\right)$	DM1	Apply limits $1 \to \sqrt{2}$.
	$\frac{\pi}{3}(4\sqrt{2}-5)$	A1	CAO, allow $\frac{\pi}{3}(2\sqrt{8}-5)$, must be in given form.
		4	
10(c)	Arc length = $\frac{1}{8}(2\pi\sqrt{2})$ or $\frac{\pi\sqrt{2}}{4}$ oe	B1	Must be exact.
	Perimeter = $\sqrt{2}+$ *their* arc length	B1 FT	Must be exact, do not allow inverse trig functions.
		2	

Question Number	Scheme	Marks
9 (a) (i)	<u>Stretch</u> parallel to the x-axis $\times\frac{1}{2}$ or <u>stretch</u> parallel to the y-axis $\times\sqrt{2}$	<u>M1</u>, A1
(ii)	<u>Translate</u> by the vector $\begin{pmatrix}0\\12\end{pmatrix}$ (or translate up by 12 (units))	<u>M1</u>, A1
		(4)
(b) (i)	$12-\sqrt{x}=\sqrt{2}\sqrt{x}$	
	$12=\left(\sqrt{2}+1\right)\sqrt{x}$	M1
	$\Rightarrow\sqrt{x}=\frac{12}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=12\left(\sqrt{2}-1\right)$ *	dM1, A1 *
Alt (i)	$12-\sqrt{x}=\sqrt{2x}\Rightarrow\left(12-\sqrt{x}\right)^2=2x\Rightarrow x+24\sqrt{x}-144=0$	M1
	$\Rightarrow\left(\sqrt{x}\right)=\frac{-24\pm\sqrt{24^2-4\times-144}}{2}=-12\pm\frac{12}{2}\sqrt{4+8}=-12\pm12\sqrt{2}$	dM1
	$\sqrt{x}>0\Rightarrow\sqrt{x}=-12+12\sqrt{2}=12\left(\sqrt{2}-1\right)$*	A1
(ii)	$\Rightarrow x=12^2\left(\sqrt{2}-1\right)^2=144\left(2+1-2\sqrt{2}\right)=144\left(3-2\sqrt{2}\right)$	M1, A1
	$y\left\{=12-\sqrt{x}=12-12\left(\sqrt{2}-1\right)\right\}=12\left(2-\sqrt{2}\right)$	B1
	Or common acceptable alt forms: $P\left(432-288\sqrt{2},\ \ 24-12\sqrt{2}\right)$	
		(6) **(10 marks)**

Question	Answer	Marks	Guidance
9(b)	[Centre is] $(-4, 3)$	B1	SOI
	Radius gradient $= \frac{(\textit{their}\ 3)-10}{(\textit{their}\ -4)-2}$ $\left[=\frac{7}{6}\right]$	*M1	Attempt at finding the gradient of the radius using $(2, 10)$ and their centre, but not P, Q, the mid-point of PR or QR.
	Either $\frac{-1}{\left(\textit{their}\ \frac{7}{6}\right)} = \frac{y-10}{x-2}$ Or $10 = \frac{-1}{\left(\textit{their}\ \frac{7}{6}\right)} \times 2 + c \Rightarrow c = ...$	DM1	OE Correct method to find the equation of the tangent using $\frac{-1}{\textit{their}\text{ radius gradient}}$ and $(2, 10)$.
	$y-10 = -\frac{6}{7}(x-2)$ or $y = \left(-\frac{6}{7}\right)x + \frac{82}{7}$	A1	OE Correct equation but not stated in the required form.
	$6x+7y-82=0$ or $82-6x-7y=0$	A1	All correct terms on one side, but condone them being in the wrong order.
	Alternative Method for Question 9(b)		
	$x^2+y^2+8x-6y-60=0$	B1	OE Equation of the circle.
	$2x+2y\frac{dy}{dx}+8-6\frac{dy}{dx}=0$ Or $\frac{dy}{dx} = \frac{-2x-8}{2}\left(85-x^2-8x-16\right)^{-\frac{1}{2}}$	*M1	Differentiate implicitly to arrive at an expression with two terms containing $\frac{dy}{dx}$. Or rearrange to make y the subject and differentiate to arrive at an expression of the form $f'(x)\times f(x)$.

Question	Answer	Marks	Guidance
9(a)	$[y=]\ \{x\}\ \{+(x-1)^{-2}\}\ \ [+c]$	B1 B1	May be unsimplified.
	Sub $x=0$, $y=3$ leading to $3 = 0 + 1 + c$	M1	Substitution into an integral, expect $c=2$.
	$y = x+(x-1)^{-2}+2$ or $f(x) = x+(x-1)^{-2}+2$	A1	$\frac{-2}{(-2)(x-1)^2}$ or $\frac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	
9(b)	[Gradient of tangent =] $f'(0) = 3$	B1	
	Equation of tangent is $y-3 = \textit{their}$ gradient at $x=0(x-0)$	M1*	Expect $y=3x+3$, normal gets M0.
	Intersection given by $3x+3 = x+(x-1)^{-2}+2$	DM1	FT *their* equation from part **(a)**.
	$2x+1 = \frac{1}{(x-1)^2} \to (2x+1)(x-1)^2-1=0$ or solve equation before given form reached and show solution ($x = 3/2$) satisfies given result	A1	WWW AG
		4	
9(c)	Substitute $x=\frac{3}{2}$ leading to $(2x+1)(x-1)^2-1$ leading to $4\times¼-1=0$. Hence $x=\frac{3}{2}$ If shown in **(b)** must be referenced here (in part **(c)**)	B1	Evaluation of each bracket must be shown. Allow $\left(\frac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2-1=0$ is acceptable.
	When $x=\frac{3}{2}$ $y=7½$	B1	
		2	

Question	Answer	Marks	Guidance
9(a)	Differentiate to obtain form $kx^2(2x^3+10)^{-\frac{1}{2}}$	M1	OE
	$3x^2(2x^3+10)^{-\frac{1}{2}}$	A1	Or unsimplified equivalent.
	Substitute $x=3$ in first derivative and evaluate to find gradient	*M1	Expect $\frac{27}{8}$. Allow if first derivative of forms $k(2x^3+10)^{\frac{-1}{2}}$, $kx(2x^3+10)^{\frac{-1}{2}}$ or $kx^2(2x^3+10)^{\frac{-1}{2}}$.
	Attempt equation of tangent at $(3,8)$ with numerical gradient	DM1	Use of gradient of the normal is DM0.
	$[\pm](27x-8y-17)=0$ or integer multiples	A1	
		5	
9(b)	State or imply volume is $\pi\int(2x^3+10)\,dx$	B1	Implied if π appears only at the end. Do not allow an unsimplified: $\pi\int\left((2x^3+10)^{1/2}\right)^2$.
	Integrate to obtain $k_1x^4+k_2x$ and evaluate using limits 1 and 3	M1	Where $k_1k_2\neq 0$.
	60π	A1	OE Allow from a correct integral and sight of limits. Allow numerical answers in the range 188-189.
		3	

Question Number	Scheme	Marks
10	$(k-1)x^6+4x^3+(k-4)=0$	
(a)	$3.5x^6+4x^3+0.5=0\Rightarrow 7x^6+8x^3+1=0$	
	$\Rightarrow\left(x^3+1\right)\left(7x^3+1\right)=0$	M1
	$\Rightarrow x^3=-1,\quad x^3=-\frac{1}{7}$	A1
	$\Rightarrow x=-1,\quad x=-\frac{1}{\sqrt[3]{7}}$	A1
		(3)
(b)	Attempts $b^2-4ac=16-4(k-1)(k-4)$	M1
	$=20k-4k^2$	A1
	Solves $b^2-4ac<0\Rightarrow 4k(5-k)<0\Rightarrow k<0,\ k>5$	dM1 A1
		(4)
		(7 marks)

Question	Answer	Marks	Guidance
10(c)(i)	$(1,\ -13)$	B1	
		1	
10(c)(ii)	$[y=]\dfrac{9}{2(x\pm3)-5}+2(x\pm3)-5\pm7$	M1	Application of $\begin{pmatrix}-3\\7\end{pmatrix}$ to the original expression for C but condone +/− sign errors.
	$[y=]-\left(\dfrac{9}{2(x\pm3)-5}+2(x\pm3)-5\pm7\right)$	M1	**SC B1** for $[y=]-\left(\dfrac{9}{2x-5}+2x-5\right)$.
	$y=-\dfrac{9}{2x+1}-2x-8$	A1	Answer must be in this format; the '$y=$' can be implied by earlier inclusion.
		3	

Question	Answer	Marks	Guidance
4	$\left[\frac{dv}{dx}\right]=(9-x)^2$	B1	Allow unsimplified forms. Allow any or no notation
	Substitute $x=4$ into *their* differentiated V,	*M1	Expect 25.
	$\frac{dx}{dt}=\frac{1}{\textit{their}\text{ derivative}}\times 3.6$ (accept $\frac{dt}{dx}=\frac{\textit{their}\text{ derivative}}{3.6}$)	M1	Correct use of the chain rule, ignore incorrect conversions at this point. Expect 0.144
	$=\frac{1}{\textit{their numerical}\text{ derivative}}\times 3.6\times\frac{100}{60}$	DM1	Correct use of the conversion factors.
	$=\frac{1}{25}\times 3.6\times\frac{100}{60}=0.24$	A1	
		5	

Question	Answer	Marks	Guidance
8(a)	$\frac{dy}{dx}=[2]\quad[-2(2x+1)^{-2}]$	B1 B1	
	$\frac{d^2y}{dx^2}=8(2x+1)^{-3}$	B1	
		3	
8(b)	Set *their* $\frac{dy}{dx}=0$ and attempt solution	M1	
	$(2x+1)^2=1\rightarrow 2x+1=(\pm)1$ or $4x^2+4x=0\rightarrow(4)x(x+1)=0$	M1	Solving as far as $x=$ ....
	$x=0$	A1	WWW. Ignore other solution.
	(0, 2)	A1	One solution only. Accept $x=0$, $y=2$ only.
	$\frac{d^2y}{dx^2}>0$ from a solution $x>-\frac{1}{2}$ hence minimum	B1	Ignore other solution. Condone arithmetic slip in value of $\frac{d^2y}{dx^2}$. *Their* $\frac{d^2y}{dx^2}$ must be of the form $k(2x+1)^{-3}$
		5	

Question	Answer	Marks	Guidance
2(a)	$(0-3)^2+(y-5)^2=40$	M1	OE. Substitute $x=0$, may use $y^2-10y-6=0$.
	$y=5\pm\sqrt{31}$	A1	OE. Must be surd form.
		2	
2(b)	$\{x^2+(y-5)^2\}=\{31\}$ Allow $(x-0)^2$	B1FT B1FT	B1 FT for *their* 5 and B1 FT for *their* 31. Don't allow surd form.
		2	

Question	Answer	Marks	Guidance
6	$\frac{\frac{10(1-r^8)}{1-r}}{\frac{10(1-r^4)}{1-r}}=\frac{17}{16}$ $\left[a\frac{(1-r^8)}{(1-r)}=\frac{17}{16}\times a\frac{(1-r^4)}{(1-r)}\right]$	M1*	OE, i.e. substituting p and q expressions into ratio $\frac{17}{16}$. $16=a\frac{(1-r^4)}{(1-r)}$, $17=a\frac{(1-r^8)}{(1-r)}$ gets M0 unless recovered later.
	Simplifying to $16r^8-17r^4+1\ [=0]$ (or equivalent form)	DM1	Or $\frac{(1-r^8)}{(1-r^4)}=(1+r^4)=\frac{17}{16}$.
	$\left[(16r^4-1)(r^4-1)=0\right]\Rightarrow r=\pm\frac{1}{2}$	A1	Or $r^4=\frac{1}{16}\ \Rightarrow\ r=\pm\frac{1}{2}$ (condone extra $r=\pm1$ solution).
	$S_\infty=\frac{10}{1-\left([\pm]\frac{1}{2}\right)}$	DM1	Use of correct sum to infinity formula with either of *their* r values providing $\lvert r\rvert<1$.
	$S_\infty=20$ and $\frac{20}{3}$	A1	Allow 6.67 or better. A0 if there is only one or more than two S_∞ values.
		5	