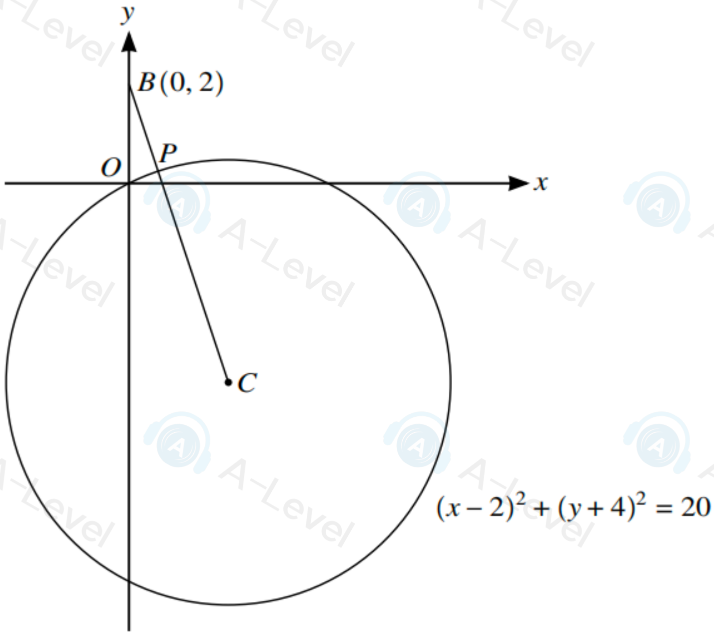
7



The diagram shows the circle with equation $(x-2)^2+(y+4)^2=20$ and with centre C. The point B has coordinates (0, 2) and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

(b) Hence find the coordinates of P, giving your answer in exact form. [5]

9.

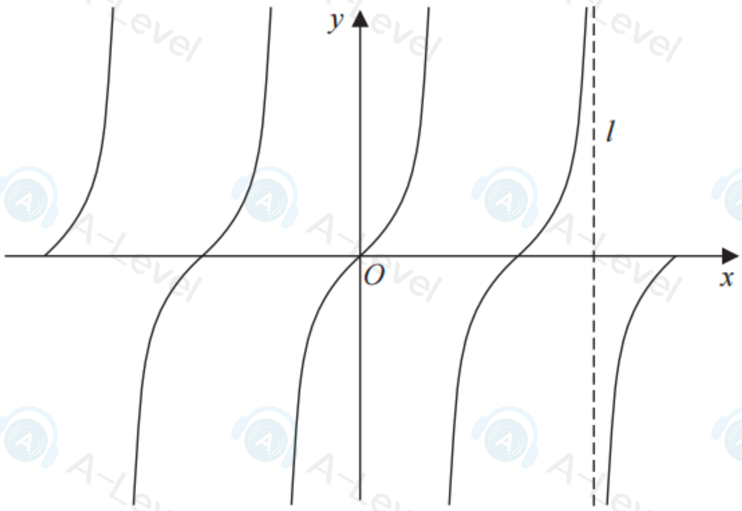


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

The line l, shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l.

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \qquad -2\pi \leqslant x \leqslant 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leqslant x \leqslant 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leqslant x \leqslant 40\pi$

(ii) $-10\pi \leqslant x \leqslant \frac{5}{2}\pi$

(2)

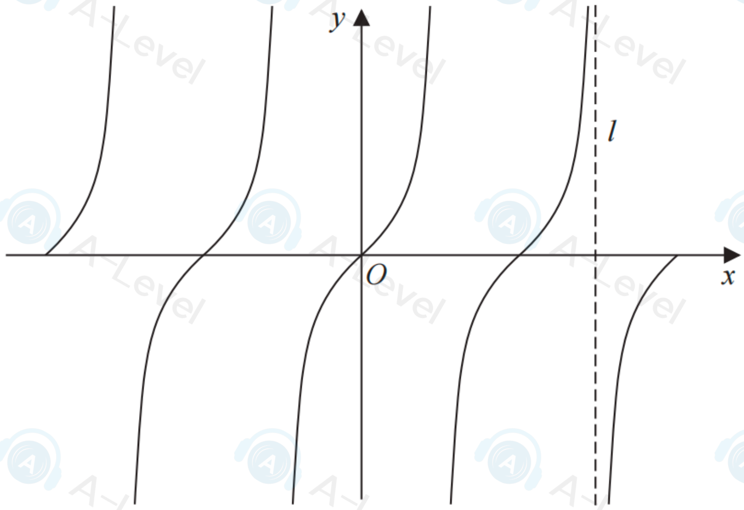


Diagram 1

7. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = \dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$
- the point $P(4, -1)$ lies on C

(a) (i) find the value of the gradient of C at P

(ii) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $f(x)$.

(6)

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$. [2]

6 The first term of a convergent geometric progression is 10. The sum of the first 4 terms of the progression is p and the sum of the first 8 terms of the progression is q. It is given that $\dfrac{q}{p} = \dfrac{17}{16}$.

Find the two possible values of the sum to infinity. [5]

3 A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

11 The function f is defined by $f(x) = 10 + 6x - x^2$ for $x \in \mathbb{R}$.

(a) By completing the square, find the range of f. [3]

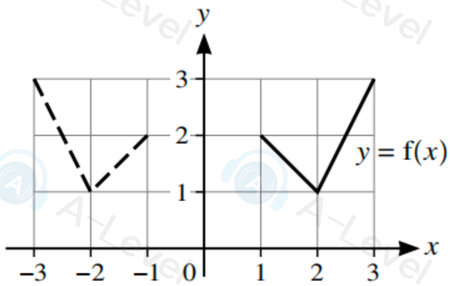
The function g is defined by $g(x) = 4x + k$ for $x \in \mathbb{R}$ where k is a constant.

(b) It is given that the graph of $y = g^{-1}f(x)$ meets the graph of $y = g(x)$ at a single point P.

Determine the coordinates of P. [6]

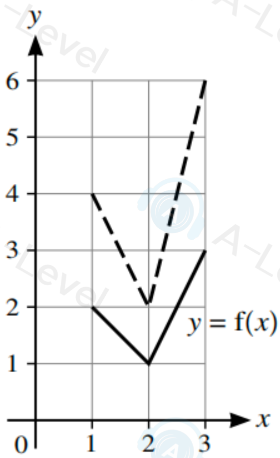
3 In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$.

(a)



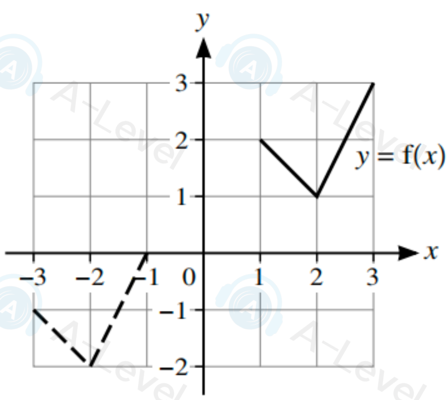
State, in terms of f, the equation of the graph shown with broken lines. [1]

(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

1 The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve has equation $y = f(x),\ x > 0$

The point $P(4, 12)$ lies on the curve.

Given that

- $f'(x) = 3\sqrt{x} + kx^2$ where k is a constant
- the equation of the tangent to the curve at P has equation $y = 10x + c$ where c is a constant

(a) (i) show that $k = \frac{1}{4}$

(ii) find the value of c

(4)

(b) Hence find the value of $f''(x)$ at P.

(3)

(c) Find $f(x)$.

(4)

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(b) Find the perimeter of the shaded region. [3]

(c) Find the area of the shaded region. [3]