

7. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = \frac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$
- the point $P(4, -1)$ lies on C

(a) (i) find the value of the gradient of C at P

(ii) Hence find the equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found.

(4)

(b) Find $f(x)$.

(6)

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

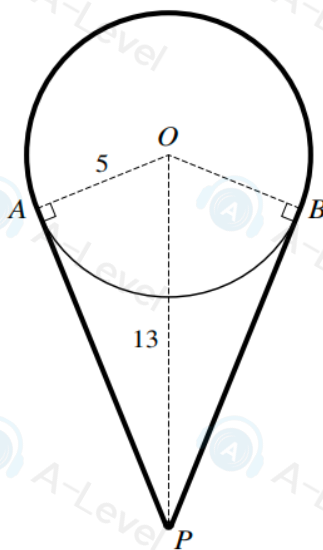
[4]

The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$.

[2]

5



The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O . Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord.

[6]

3

(a) Find the coefficients of x^3 and x^4 in the expansion of $(3 - ax)^5$, where a is a constant. Give your answers in terms of a .

[3]

(b) Given that the coefficient of x^4 in the expansion of $(ax + 7)(3 - ax)^5$ is 240, find the positive value of a .

[3]

10. A curve has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = ax - 12x^{\frac{1}{3}}$, where a is a constant
- $f''(x) = 0$ when $x = 27$
- the curve passes through the point $(1, -8)$

(a) find the value of a .

(3)

(b) Hence find $f(x)$.

(4)

It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

(b) Find $f(x)$.

[4]

(c) Find the coordinates of the other stationary point and determine its nature.

[4]

3. The share price of a company is monitored.

Exactly 3 years after monitoring began, the share price was £1.05

Exactly 5 years after monitoring began, the share price was £1.65

The share price, £ V , of the company is modelled by the equation

$$V = pt + q$$

where t is the number of years after monitoring began and p and q are constants.

(a) Find the value of p and the value of q .

(3)

Exactly T years after monitoring began, the share price was £2.50

(b) Find the value of T , according to the model, giving your answer to one decimal place.

(2)

(b) Find the first three terms in ascending powers of x of the expansion of $(1 - 3x)^4$.

[2]

(c) Hence find the coefficient of x^2 in the expansion of $(1 + 2x)^5(1 - 3x)^4$.

[2]

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6. The point A has coordinates $(-4, 11)$ and the point B has coordinates $(8, 2)$.

(a) Find the gradient of the line AB , giving your answer as a fully simplified fraction. (2)

The point M is the midpoint of AB . The line l passes through M and is perpendicular to AB .

(b) Find an equation for l , giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found. (4)

The point C lies on l such that the area of triangle ABC is 37.5 square units.

(c) Find the two possible pairs of coordinates of point C . (5)

(b) Show that $\frac{2}{3} + \frac{2}{3(3x-1)}$ can be expressed as $\frac{2x}{3x-1}$. [2]

(c) State the range of f . [1]

6 It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\frac{1}{\sin \alpha} + \frac{1}{\tan \alpha}$. [5]

1 A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$.

Find $f(x)$. [3]

11 The coordinates of points A, B and C are $A(5, -2), B(10, 3)$ and $C(2p, p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p . [3]

(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p . [4]

(ii) Find the equation of the circle which passes through A, B and C , giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [4]