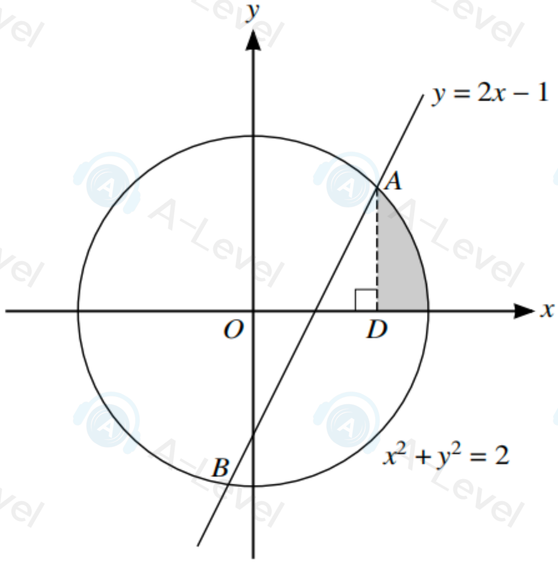

The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A. [4]

(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}\left(b\sqrt{c} - d\right)$, where a, b, c and d are integers. [4]

(c) Find an exact expression for the perimeter of the shaded region. [2]

A-Level

9.

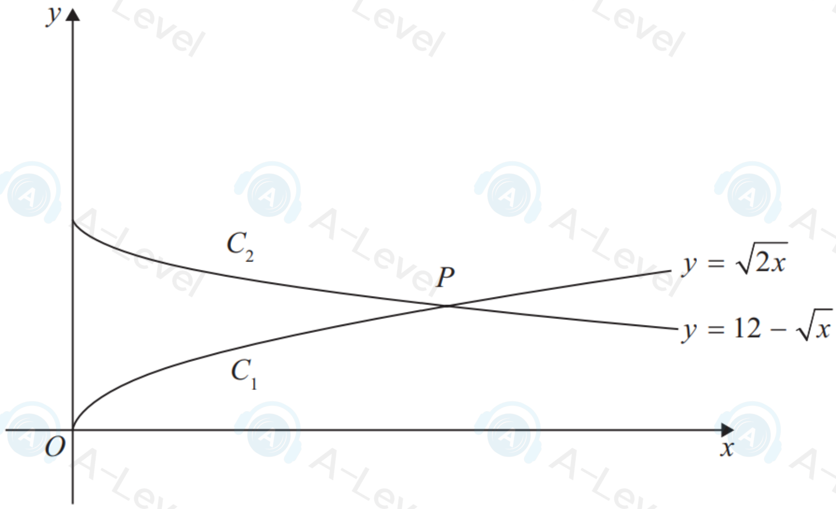

Figure 4

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 4 shows a sketch of

- the graph C_1 with equation $y = \sqrt{2x}$
- the graph C_2 with equation $y = 12 - \sqrt{x}$

(a) Describe fully the single transformation that would transform

(i) the graph with equation $y = \sqrt{x}$ onto C_1

(ii) the graph with equation $y = -\sqrt{x}$ onto C_2

(4)

The graphs C_1 and C_2 meet at the point P, as shown in Figure 4.

(b) (i) Show that the x coordinate of P is a solution of

$$\sqrt{x} = 12\left(\sqrt{2} - 1\right)$$

(ii) Hence find, in simplest form, the exact coordinates of P.

(6)

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9. (i)

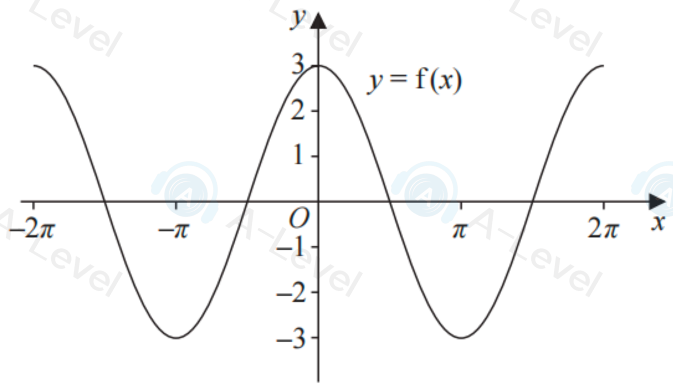

Figure 3

Figure 3 shows part of the graph of the trigonometric function with equation $y = \mathrm{f}(x)$

(a) Write down an expression for $\mathrm{f}(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = \mathrm{f}\left(x + \frac{\pi}{4}\right)$

Show clearly the coordinates of all the points where the curve intersects the coordinate axes.

(3)

(ii)

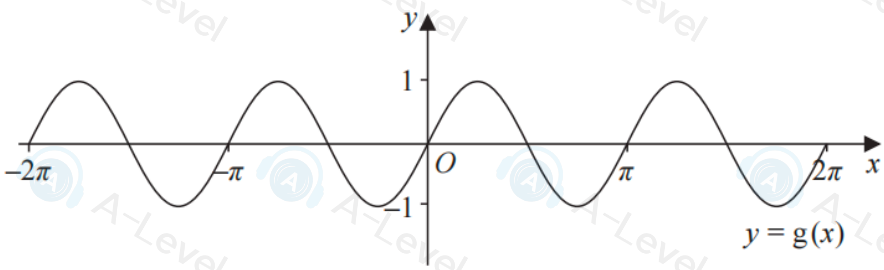

Figure 4

Figure 4 shows part of the graph of the trigonometric function with equation $y = \mathrm{g}(x)$

(a) Write down an expression for $\mathrm{g}(x)$

(2)

On a separate diagram,

(b) sketch, for $-2\pi < x < 2\pi$, the graph of the curve with equation $y = \mathrm{g}(x) - 2$

Show clearly the coordinates of the y intercept.

(2)

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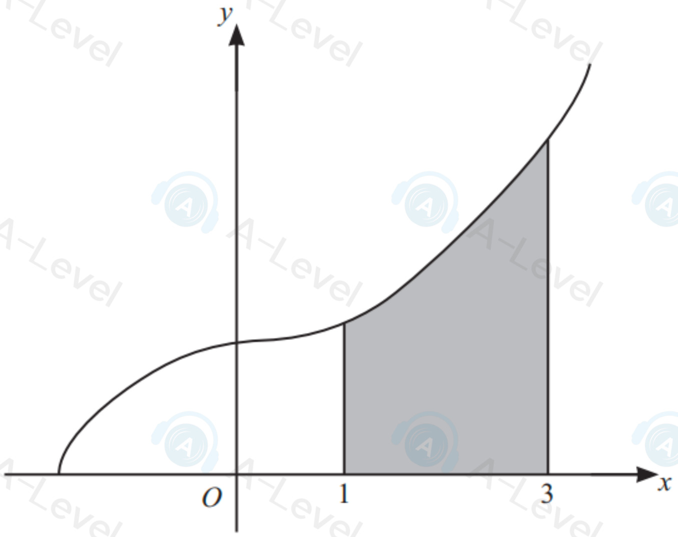
The tangent to the curve at (0, 3) intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation $(2x + 1)(x - 1)^2 - 1 = 0$. [4]

(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

9



The diagram shows the curve with equation $y = \sqrt{2x^3 + 10}$.

(a) Find the equation of the tangent to the curve at the point where $x = 3$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [5]

(b) The region shaded in the diagram is enclosed by the curve and the straight lines $x = 1$, $x = 3$ and $y = 0$.

Find the volume of the solid obtained when the shaded region is rotated through 360° about the x-axis. [3]

10. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$$(k-1)x^6 + 4x^3 + (k-4) = 0 \qquad \text{where } k \text{ is a constant}$$

(a) Find the exact solutions to the given equation for $k = 4.5$ **(3)**

(b) Find the set of possible values of k for which the given equation has no real roots. **(4)**

(b) Find $\dfrac{d^2y}{dx^2}$ and hence determine the nature of each stationary point. [3]

(c) The curve C is transformed to the curve C_1 using a translation of $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$ followed by reflection in the x-axis.

(i) State the coordinates of the maximum point of C_1. [1]

(ii) Find the equation of C_1 in the form $y = \dfrac{a}{bx+c} + dx + e$, where a, b, c, d and e are integers. [3]

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4 A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute. [5]

8. The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

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(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

2. Given that

$$a = \frac{1}{64}x^2 \qquad b = \frac{16}{\sqrt{x}}$$

express each of the following in the form kx^n where k and n are simplified constants.

(a) $a^{\frac{1}{2}}$

(1)

(b) $\dfrac{16}{b^3}$

(1)

(c) $\left(\dfrac{ab}{2}\right)^{-\frac{4}{3}}$

(2)

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(b) Find the equation of the circle which has AB as its diameter. [2]

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6 The first term of a convergent geometric progression is 10. The sum of the first 4 terms of the progression is p and the sum of the first 8 terms of the progression is q. It is given that $\frac{q}{p} = \frac{17}{16}$.

Find the two possible values of the sum to infinity. [5]