6 A curve passes through the point $\left(\frac{4}{5}, -3\right)$ and is such that $\frac{dy}{dx} = \frac{-20}{(5x-3)^2}$.

(a) Find the equation of the curve. [4]

(b) The curve is transformed by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by a translation of $\begin{pmatrix} 2 \\ 10 \end{pmatrix}$.

Find the equation of the new curve. [3]

10.

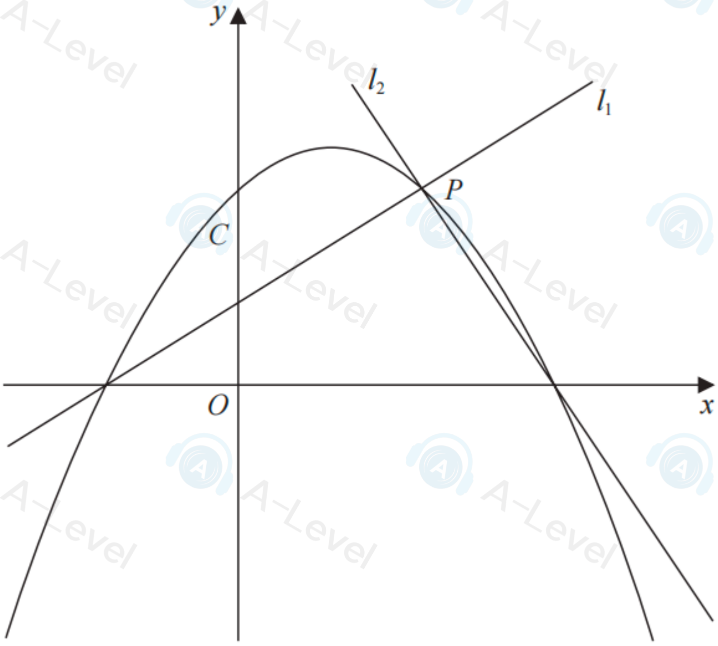


Figure 5

Figure 5 shows a sketch of the quadratic curve C with equation

$$y = -\frac{1}{4}(x+2)(x-b) \qquad \text{where } b \text{ is a positive constant}$$

The line l_1 also shown in Figure 5,

- has gradient $\frac{1}{2}$
- intersects C on the negative x-axis and at the point P

(a) (i) Write down an equation for l_1 **(1)**

(ii) Find, in terms of b, the coordinates of P **(3)**

Given that the line l_2 is perpendicular to l_1 and intersects C on the positive x-axis,

(b) find, in terms of b, an equation for l_2 **(2)**

Given also that l_2 intersects C at the point P

(c) show that another equation for l_2 is

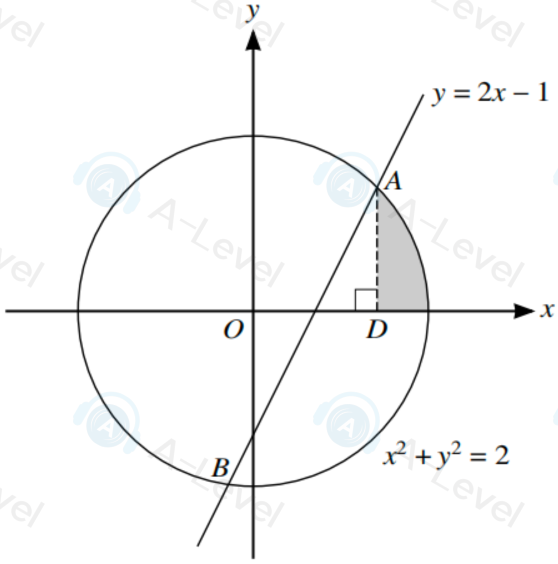
$$y = -2x + \frac{5b}{2} - 4$$

(2)

(d) Hence, or otherwise, find the value of b **(2)**

(b) Verify that the line AB is the normal to the curve at A. [3]

(c) Find the area of the shaded region. [5]



The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A. [4]

(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}(b\sqrt{c} - d)$, where a, b, c and d are integers. [4]

(c) Find an exact expression for the perimeter of the shaded region. [2]

11.

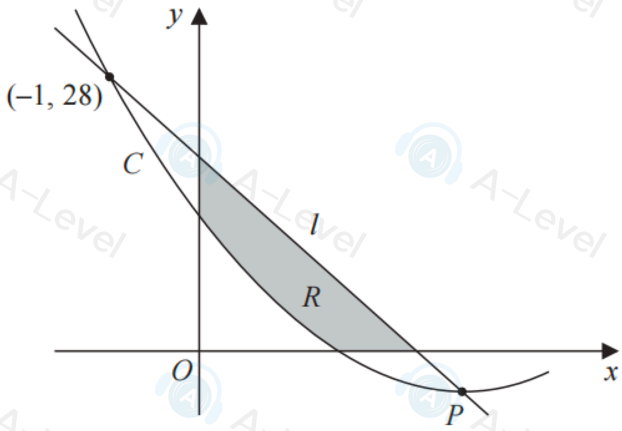


Figure 5

Figure 5 shows part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(-1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)

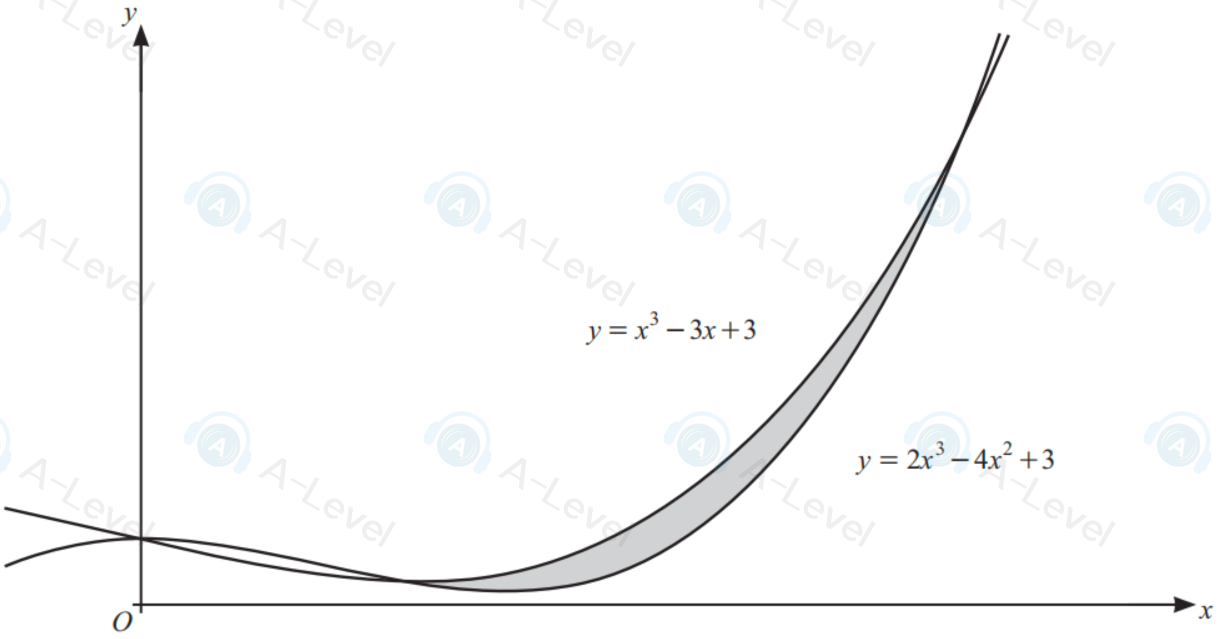
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE

(b) Find the exact value of the area of the shaded region. [6]

9



The diagram shows the curves with equations $y = x^3 - 3x + 3$ and $y = 2x^3 - 4x^2 + 3$.

(a) Find the x-coordinates of the points of intersection of the curves. [3]

(b) Find the area of the shaded region. [4]

3 Given that $\int_1^3 \left(\frac{a}{(4x-3)^2} + 2 \right) dx = 12$, find the value of the constant a. [4]

10 A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2} \right) dx = \frac{13}{12}$.

Find the value of k. [5]