8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the equation of the tangent to the curve with equation

$$y = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

at the point $P(4, 12)$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(5)

The curve with equation $y = f(x)$ also passes through the point $P(4, 12)$

Given that

$$f'(x) = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

(b) find $f(x)$ giving the coefficients in simplest form.

(5)

6 The first term of a convergent geometric progression is 10. The sum of the first 4 terms of the progression is p and the sum of the first 8 terms of the progression is q. It is given that $\frac{q}{p} = \frac{17}{16}$.

Find the two possible values of the sum to infinity. [5]

5.

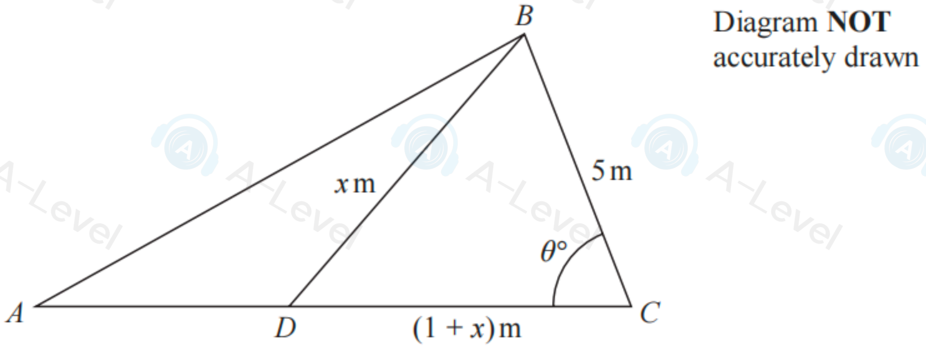

Figure 2

Figure 2 shows the plan view of a frame for a flat roof.

The shape of the frame consists of triangle *ABD* joined to triangle *BCD*.

Given that

- $BD = x$ m
- $CD = (1 + x)$ m
- $BC = 5$ m
- angle $BCD = \theta°$

(a) show that $\cos\theta° = \dfrac{13 + x}{5 + 5x}$

(2)

Given also that

- $x = 2\sqrt{3}$
- angle $BAC = 30°$
- *ADC* is a straight line

(b) find the area of triangle *ABC*, giving your answer, in m^2, to one decimal place.

(5)

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(b) One of the values of p found in **(a)** is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

4.

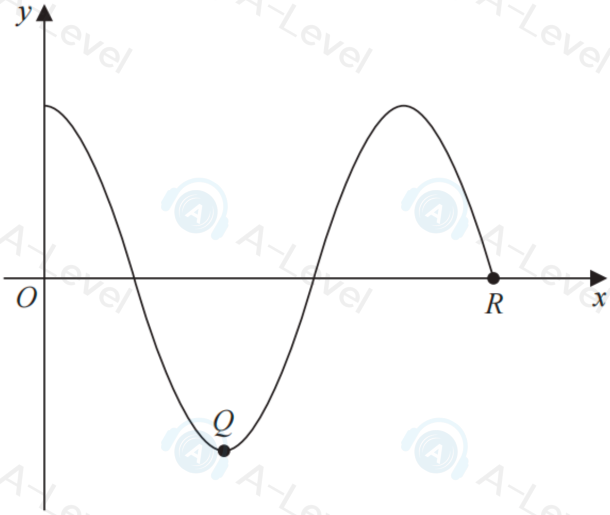

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \text{f}(x)$, where

$$\text{f}(x) = \cos 2x^{\circ} \qquad 0 \leqslant x \leqslant k$$

The point Q and the point $R(k, 0)$ lie on the curve and are shown in Figure 2.

(a) State

(i) the coordinates of Q,

(ii) the value of k.

(3)

(b) Given that there are exactly two solutions to the equation

$$\cos 2x^{\circ} = p \qquad \text{in the region } 0 \leqslant x \leqslant k$$

find the range of possible values for p.

(2)

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It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k. [3]

9.

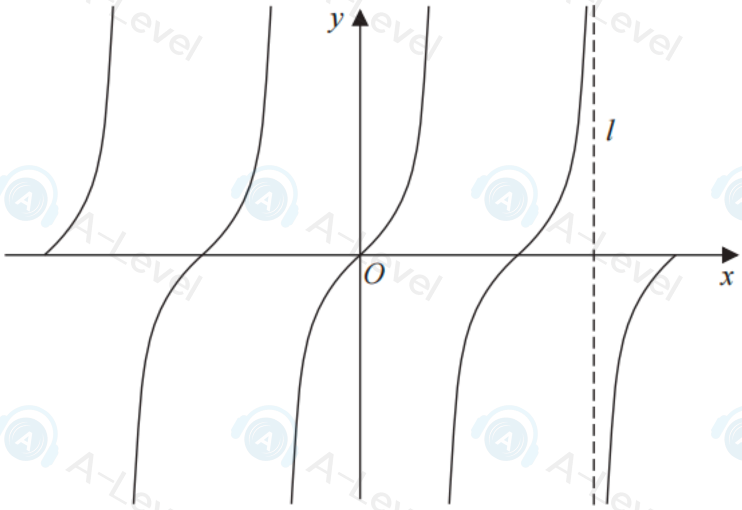

Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

The line l, shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l.

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \qquad -2\pi \leqslant x \leqslant 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leqslant x \leqslant 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leqslant x \leqslant 40\pi$

(ii) $-10\pi \leqslant x \leqslant \frac{5}{2}\pi$

(2)

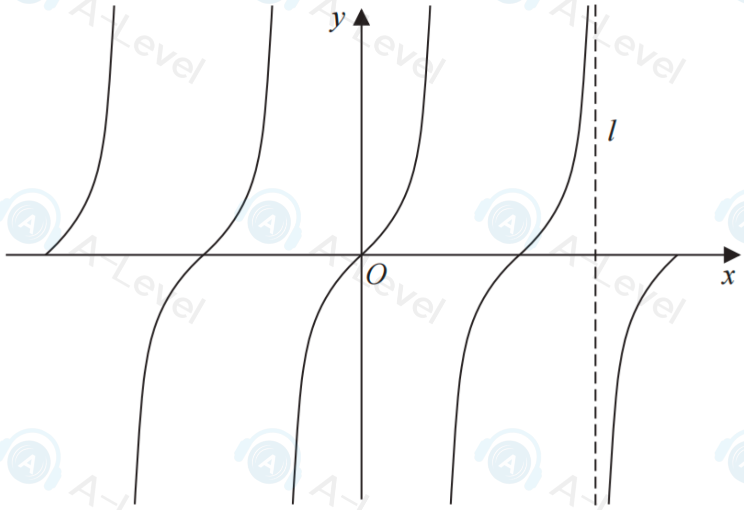

Diagram 1

8 The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

2 The first two terms of a geometric progression are

$$4\sin^2\theta, \quad 8\sin^3\theta,$$

where θ is an angle such that $0 < \theta < \frac{1}{6}\pi$.

Given that the sum to infinity of the progression is $\frac{1}{2}$, find the value of θ. Give your answer in the form $\sin^{-1} k$, where k is a rational number. [4]