blank

8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

(a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point P is the minimum point of C_1

(b) Deduce the coordinates of P.

(1)

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A, B, C and D are constants.

Given that C_2

- passes through P
- intersects the x-axis at -4, -2 and 3

(c) find, making your method clear, the values of A, B, C and D.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$. [3]

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x)$, $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{12}{\sqrt{x}} + \dfrac{x}{3} - 4$
- the point $P(9, 8)$ lies on C

(a) find, in simplest form, $\mathrm{f}(x)$

(5)

The line l is the normal to C at P

(b) Find the coordinates of the point at which l crosses the y-axis.

(4)

DO NOT WRITE IN THIS AREA

(b) State the radius of the original circle. [1]

(c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

(d) State the coordinates of the centre of the original circle. [2]

blank

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = f(x)$ where

$$f(x) = 2(x+1)(x-3)^2$$

(a) Sketch a graph of C.

Show on your graph the coordinates of the points where C cuts or meets the coordinate axes.

(3)

(b) Write f(x) in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

(c) Hence, find the equation of the tangent to C at the point where $x = \frac{1}{3}$

(4)

DO NOT WRITE IN THIS AREA

The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

(c) State the maximum possible value of a for which fg can be formed. [1]

(d) Assuming that fg can be formed, find and simplify an expression for fg(x). [2]

6 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of f(x) and g(x) in completed square form, express g(x) in the form $f(x+p)+q$, where p and q are constants. [4]

(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

7 **(a)** Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

(b) Hence solve the equation $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

6 The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express f(x) in the form $2(x+a)^2 + b$. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$. [2]

8 **(a)** Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geqslant k$, where k is a constant.

(b) Find the least value of k for which the function f^{-1} exists. [1]

For the rest of this question, you should assume that k has the value found in part **(b)**.

(c) Find an expression for $f^{-1}(x)$. [3]

(d) Hence or otherwise solve the equation $ff(x) = 29$. [3]

6 A curve passes through the point $\left(\frac{4}{5}, -3\right)$ and is such that $\frac{dy}{dx} = \frac{-20}{(5x-3)^2}$.

(a) Find the equation of the curve. [4]

(b) The curve is transformed by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by a translation of $\begin{pmatrix} 2 \\ 10 \end{pmatrix}$.

Find the equation of the new curve. [3]

6 It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\frac{1}{\sin\alpha} + \frac{1}{\tan\alpha}$. [5]