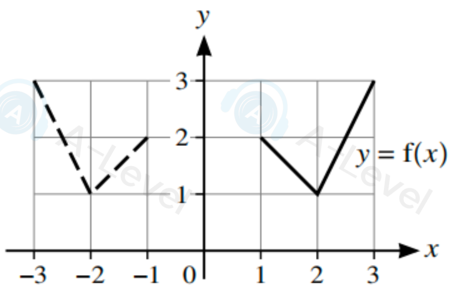
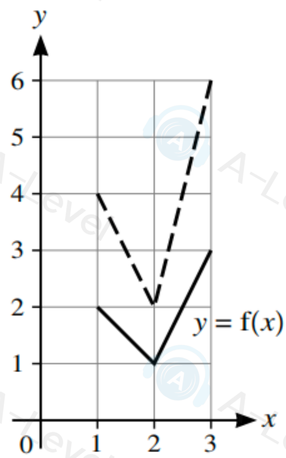
**3** In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = \mathrm{f}(x)$. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

**(a)**



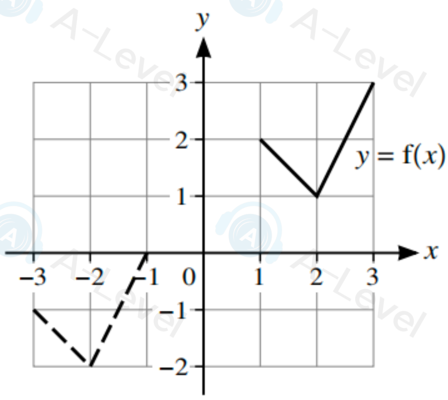

State, in terms of f, the equation of the graph shown with broken lines. [1]

**(b)**




State, in terms of f, the equation of the graph shown with broken lines. [1]

**(c)**




State, in terms of f, the equation of the graph shown with broken lines. [2]

**8** **(a)** Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where $a$ and $b$ are constants to be found. [2]

The function $\mathrm{f}(x) = 3x^2 - 12x + 14$ is defined for $x \geqslant k$, where $k$ is a constant.

**(b)** Find the least value of $k$ for which the function $\mathrm{f}^{-1}$ exists. [1]

For the rest of this question, you should assume that $k$ has the value found in part **(b)**.

**(c)** Find an expression for $f^{-1}(x)$. [3]

**(d)** Hence or otherwise solve the equation $ff(x) = 29$. [3]

**5.** **In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.**

The curve $C$ has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = \dfrac{12}{\sqrt{x}} + \dfrac{x}{3} - 4$
- the point $P(9, 8)$ lies on $C$

(a) find, in simplest form, f($x$)

**(5)**

The line $l$ is the normal to $C$ at $P$

(b) Find the coordinates of the point at which $l$ crosses the $y$-axis.

**(4)**

**(b)** State the radius of the original circle. [1]

**(c)** State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

**(d)** State the coordinates of the centre of the original circle. [2]

**6** Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

**(a)** By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where $p$ and $q$ are constants. [4]

**(b)** Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

**11** The function f is defined by $f(x) = 10 + 6x - x^2$ for $x \in \mathbb{R}$.

**(a)** By completing the square, find the range of f. [3]

The function g is defined by $g(x) = 4x + k$ for $x \in \mathbb{R}$ where $k$ is a constant.

**(b)** It is given that the graph of $y = g^{-1}f(x)$ meets the graph of $y = g(x)$ at a single point $P$.

Determine the coordinates of $P$. [6]

**9** The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

**(a)** Express $f(x)$ in the form $(x-a)^2 + b$. [2]

It is given that f is a one-one function.

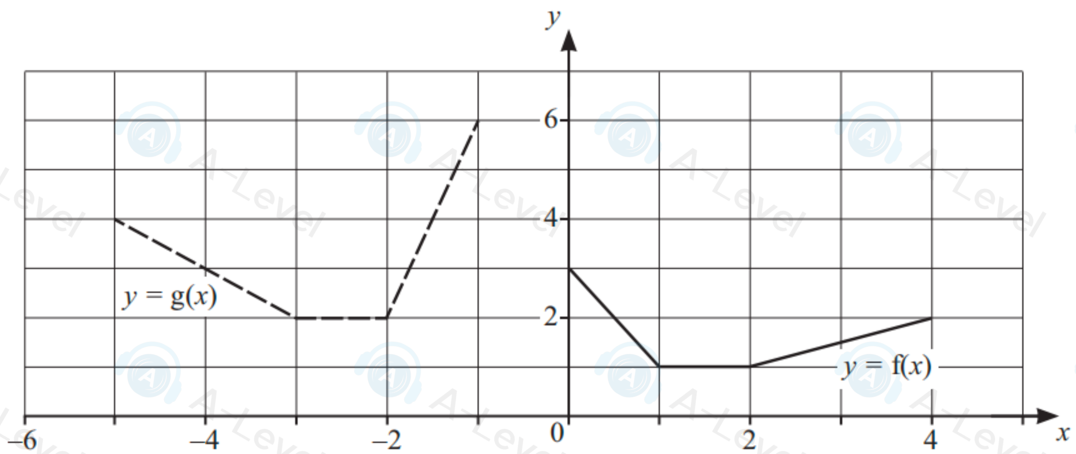
**(b)** State the smallest possible value of $c$. [1]

It is now given that $c = 5$.

**(c)** Find an expression for $f^{-1}(x)$ and state the domain of $f^{-1}$. [3]

**(d)** Find an expression for $gf(x)$ and state the range of gf. [3]

**5**




In the diagram, the graph with equation $y = f(x)$ is shown with solid lines and the graph with equation $y = g(x)$ is shown with broken lines.

**(a)** Describe fully a sequence of three transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [6]

**(b)** Find an expression for $g(x)$ in the form $af(bx+c)$, where $a$, $b$ and $c$ are integers. [2]

7. The curve $C$ has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = \dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$
- the point $P(4, -1)$ lies on $C$

(a) (i) find the value of the gradient of $C$ at $P$

(ii) Hence find the equation of the normal to $C$ at $P$, giving your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers to be found.

**(4)**

(b) Find $f(x)$.

**(6)**

DO NOT WRITE IN THIS AREA

**(b)** Obtain an expression for $f^{-1}(x)$ and state the domain of $f^{-1}$. [4]

The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

**(c)** Obtain a simplified expression for $gf(x)$. [2]

blank

8. The curve $C_1$ has equation

$$y = 3x^2 + 6x + 9$$

(a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where $a$, $b$ and $c$ are constants to be found.

**(3)**

The point $P$ is the minimum point of $C_1$

(b) Deduce the coordinates of $P$.

**(1)**

A different curve $C_2$ has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where $A$, $B$, $C$ and $D$ are constants.

Given that $C_2$

- passes through $P$
- intersects the $x$-axis at $-4$, $-2$ and 3

(c) find, making your method clear, the values of $A$, $B$, $C$ and $D$.

**(5)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS A

**(b)** Find an expression for $(fg)^{-1}(x)$. [3]