

1 Solve the equation $8 \sin^2 \theta + 6 \cos \theta + 1 = 0$ for $0^\circ < \theta < 180^\circ$.

[3]

9.

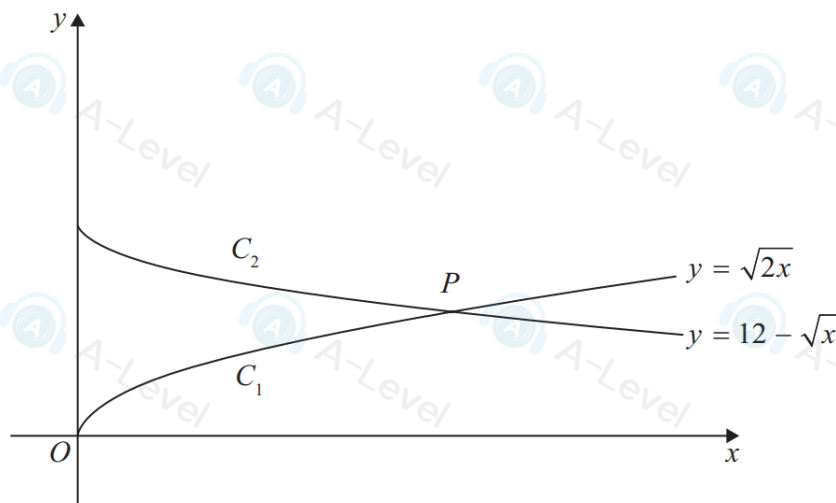


Figure 4

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 4 shows a sketch of

- the graph C_1 with equation $y = \sqrt{2x}$
- the graph C_2 with equation $y = 12 - \sqrt{x}$

(a) Describe fully the single transformation that would transform

- (i) the graph with equation $y = \sqrt{x}$ onto C_1
- (ii) the graph with equation $y = -\sqrt{x}$ onto C_2

(4)

The graphs C_1 and C_2 meet at the point P , as shown in Figure 4.

(b) (i) Show that the x coordinate of P is a solution of

$$\sqrt{x} = 12(\sqrt{2} - 1)$$

(ii) Hence find, in simplest form, the exact coordinates of P .

(6)

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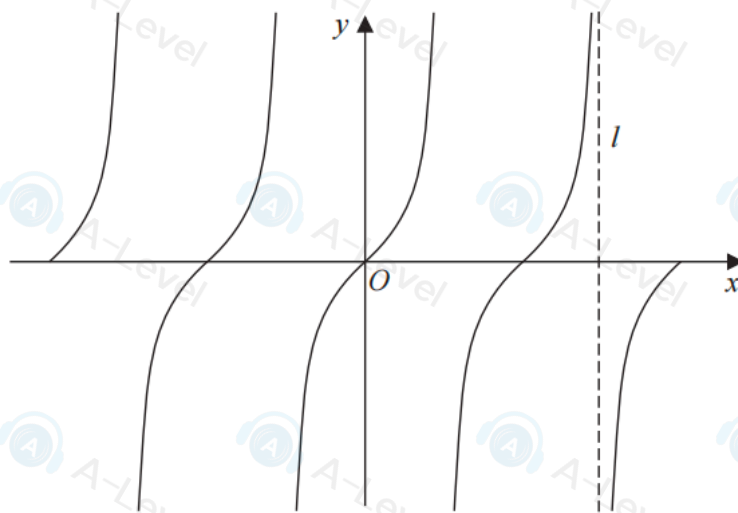


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \quad -2\pi \leq x \leq 2\pi$$

The line l , shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l .

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \quad -2\pi \leq x \leq 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leq x \leq 2\pi$

(4)

(c) State the number of solutions of the equation $\tan x = \frac{1}{x} + 1$ in the region

(i) $0 \leq x \leq 40\pi$

(ii) $-10\pi \leq x \leq \frac{5}{2}\pi$

(2)

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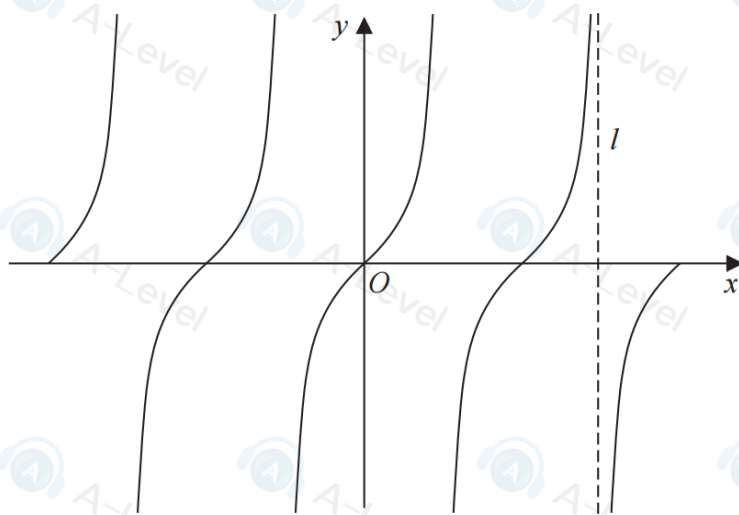


Diagram 1

8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

- (a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where a , b and c are constants to be found.

(3)

The point P is the minimum point of C_1

- (b) Deduce the coordinates of P .

(1)

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A , B , C and D are constants.

Given that C_2

- passes through P
- intersects the x -axis at -4 , -2 and 3

- (c) find, making your method clear, the values of A , B , C and D .

(5)

- (b) Find an expression for $(fg)^{-1}(x)$.

[3]

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10.

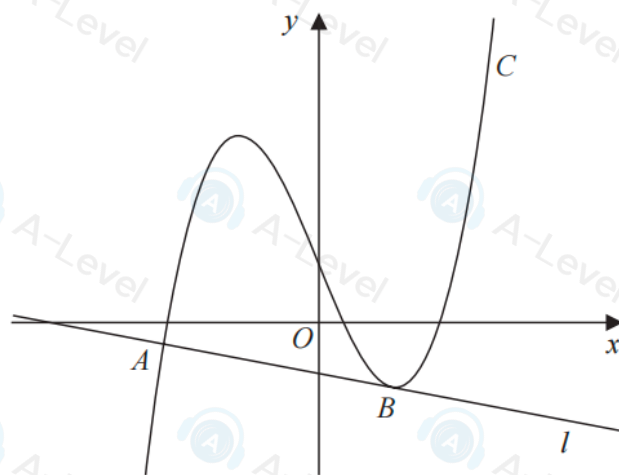


Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where k is a constant.

- (a) Find $\frac{dy}{dx}$

(2)

The line l , shown in Figure 5, is the normal to C at the point A with x coordinate $-\frac{7}{2}$

Given that l is also a tangent to C at the point B ,

- (b) show that the x coordinate of the point B is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

- (c) Hence find the x coordinate of B , justifying your answer.

(2)

Given that the y intercept of l is -1

- (d) find the value of k .

(4)

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A , its y -coordinate is **decreasing** at the rate of k units per second and its x -coordinate is **increasing** at the rate of k units per second.

- (b) Find the coordinates of A .

[6]

8.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$xy = \frac{15}{2} - 5x \quad x \neq 0$$

The curve C_2 has equation

$$y = x^3 - \frac{7}{2}x - 5$$

(a) Show that C_1 and C_2 meet when

$$2x^4 - 7x^2 - 15 = 0 \quad (2)$$

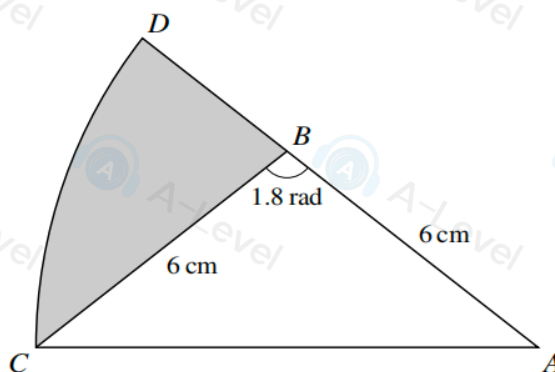
Given that C_1 and C_2 meet at points P and Q (b) find, using algebra, the exact distance PQ (5)

Another curve, with equation $y = f(x)$, has a single stationary point at the point (p, q) , where p and q are constants. This curve is transformed to a curve with equation

$$y = -3f\left(\frac{1}{4}(x+8)\right).$$

(b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q . [3]

9



The diagram shows triangle ABC with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc CD is part of a circle with centre A and ABD is a straight line.

(a) Find the perimeter of the shaded region. [5](b) Find the area of the shaded region. [3]10 Points $A(-2, 3)$, $B(3, 0)$ and $C(6, 5)$ lie on the circumference of a circle with centre D .(a) Show that angle $ABC = 90^\circ$. [2]

(b) Hence state the coordinates of D .

[1]

(c) Find an equation of the circle.

[2]

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E .

[5]

1 The coefficient of x^3 in the expansion of $\left(p + \frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant p .

[4]