10. A curve has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = ax - 12x^{\frac{1}{3}}$, where a is a constant
- $f''(x) = 0$ when $x = 27$
- the curve passes through the point $(1, -8)$

(a) find the value of a.

(3)

(b) Hence find $f(x)$.

(4)

It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

(b) Find $f(x)$. [4]

(c) Find the coordinates of the other stationary point and determine its nature. [4]

3 Given that $\int_1^3 \left(\frac{a}{(4x-3)^2} + 2\right) dx = 12$, find the value of the constant a. [4]

1 A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$.

Find $f(x)$. [3]

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3. (i) Solve

$$\frac{3}{x} > 4$$

(3)

(ii)

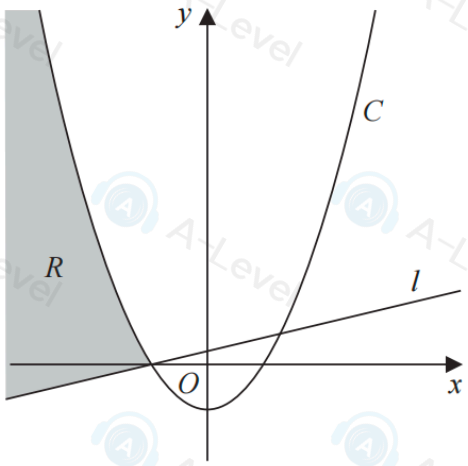

Figure 1

Figure 1 shows a sketch of the curve C and the straight line l.

The infinite region R, shown shaded in Figure 1, lies in quadrants 2 and 3 and is bounded by C and l only.

Given that

- l has a gradient of 3
- C has equation $y = 2x^2 - 50$
- C and l intersect on the negative x-axis

use inequalities to define the region R.

(3)

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

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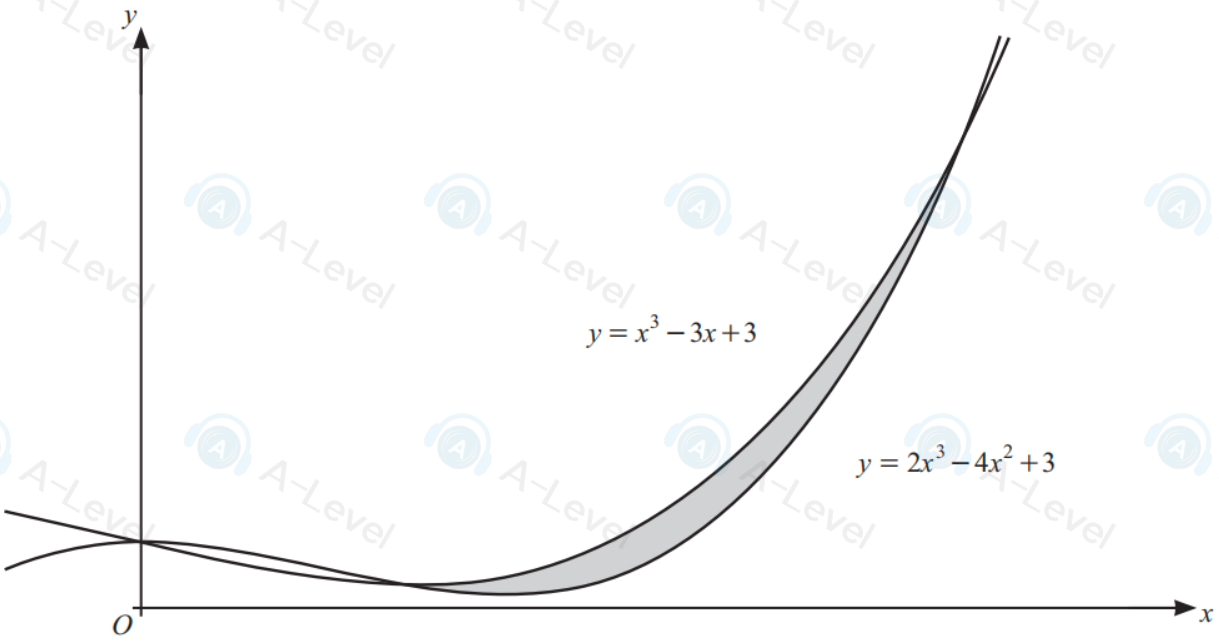

The diagram shows the curves with equations $y = x^3 - 3x + 3$ and $y = 2x^3 - 4x^2 + 3$.

(a) Find the x-coordinates of the points of intersection of the curves. [3]

(b) Find the area of the shaded region. [4]

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10.

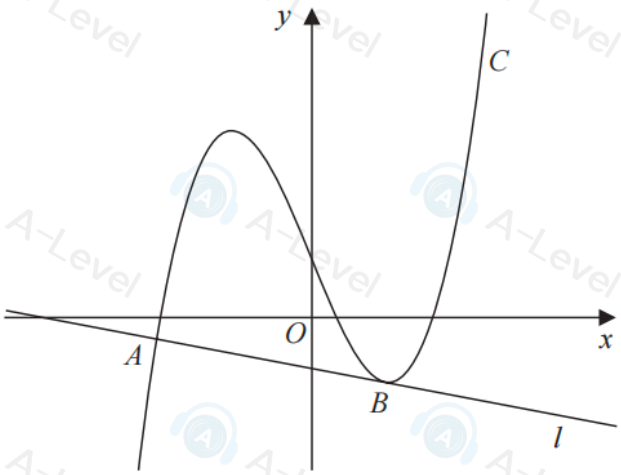

Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where k is a constant.

(a) Find $\frac{dy}{dx}$

(2)

The line l, shown in Figure 5, is the normal to C at the point A with x coordinate $-\frac{7}{2}$

Given that l is also a tangent to C at the point B,

(b) show that the x coordinate of the point B is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

(c) Hence find the x coordinate of B, justifying your answer.

(2)

Given that the y intercept of l is -1

(d) find the value of k.

(4)

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A, its y-coordinate is **decreasing** at the rate of k units per second and its x-coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A. [6]

6 A curve passes through the point $\left(\frac{4}{5}, -3\right)$ and is such that $\frac{dy}{dx} = \frac{-20}{(5x-3)^2}$.

(a) Find the equation of the curve. [4]

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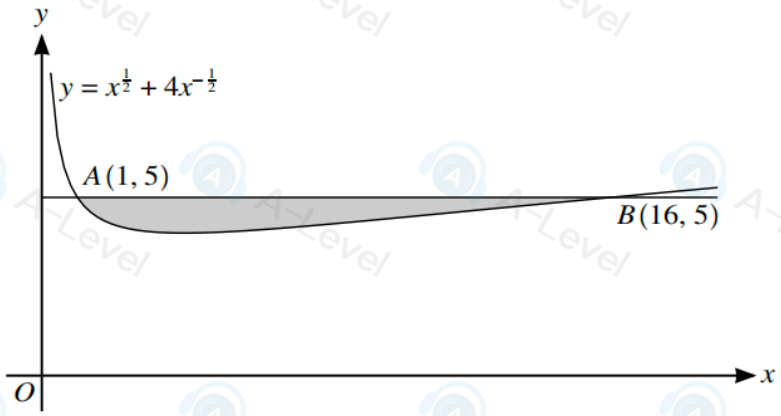
(b) The curve is transformed by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by a translation of $\begin{pmatrix} 2 \\ 10 \end{pmatrix}$.

Find the equation of the new curve. [3]

4 A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute. [5]

8



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A\,(1,\ 5)$ and $B\,(16,\ 5)$.

(a) Find the equation of the tangent to the curve at the point A. [4]

(b) Calculate the area of the shaded region. [4]