| Question | Answer | Marks | Guidance |
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| 5(a) | Use correct product rule | M1 | $\frac{d}{dx}(x^2)\cos(3x) + x^2\frac{d}{dx}(\cos 3x)$. |
| | Obtain correct derivative in any form | A1 | e.g. $2x\cos 3x - 3x^2\sin 3x$ . |
| | Equate derivative to zero and obtain $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$. | A1 | AG Condone $a = \frac{1}{3}\tan^{-1}\frac{2}{3a}$ .<br>Must at least reach expression $2x = 3x^2\tan(3x)$ or better before final answer to gain A1.<br>Final answer must be in terms of $a$. Can work with $x$ and switch to $a$ at very end.<br>Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0. |
| | | 3 | |
| 5(b) | Use the iterative process $a_{n+1} = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations | M1 | Degrees 0/3. |
| | Obtain final answer 0.36 | A1 | Must be 2d.p. |
| | Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval $(0.355, 0.365)$ | A1 | Allow small errors in 4th d.p.<br>Allow errors at start if self corrects later. |
| | 0.5 0.4 0.3 0.2 0.1 π/6 π/12<br>0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989<br>0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439<br>0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649<br>0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567<br>0.3578 0.3604 0.3614 0.3576<br>0.3580 | 3 | |
| 9(a) | Obtain $\frac{dV}{dt} = [\pm]\frac{k}{t}$ or $\frac{dV}{dt} = [\pm]\frac{1}{kt}$ | B1 | |
| | Obtain $\frac{dV}{dx} = 20x - 3x^2$ | B1 | |
| | Correct use of chain rule involving $k$ | M1 | Use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$.<br>Expressions for $\frac{dV}{dt}$ and $\frac{dV}{dx}$ must be seen to get M1. |
| | Obtain $\frac{dx}{dt} = [\pm]\frac{k}{t(20x - 3x^2)}$ or equivalent, | A1 | If this expression is first seen with numerical values, allow A1 when their value of $k$ is substituted back into the general expression. |
| | Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = -\frac{20}{37}$ to obtain given answer which must be stated<br>$\frac{dx}{dt} = -\frac{20}{37}$ needed to score final A1 | A1 | $\frac{dx}{dt} = \frac{-1}{2t(20x - 3x^2)}$ AG<br>Need to at least see $-\frac{20}{37} = \frac{k}{\frac{1}{10}\left(10 - \frac{3}{4}\right)}$ if $\frac{k}{t}$<br>or $-\frac{20}{37} = \frac{-k}{\frac{1}{10}\left(10 - \frac{3}{4}\right)}$ if $-\frac{k}{t}$ in working for correct $k$.<br>$\frac{dx}{dt} = \frac{20}{37}$ seen anywhere, then A0. |

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| 9(b) | Separate variables correctly & integrate at least one side correctly | B1 | |
| | Obtain terms $10x^2 - x^3$ | B1 | May see $-10x^2 + x^3$ if negative sign moved across or e.g. $20x^2 - 2x^3$ if 2 moved across. Allow $\frac{20x^2}{2} - \frac{3x^3}{3}$. |
| | Obtain term $\ln t$ with 'correct' coefficient from their separation of variables, for example $a \ln t$ for $\frac{a}{t}$. | B1FT | FT sign and position of 2 from their separation but B0 if error from later manipulation. |
| | Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ to evaluate a constant or as limits in a solution containing terms of the form $x^2$, $x^3$ and $\ln t$ (or $\ln 2t$) | M1 | Allow numerical and sign errors and decimals. Allow if exponentiate before substitution, even if exponentiation done incorrectly, allow for $c$ or $e^c$. |
| | Obtain correct answer in any form, for example $10x^2 - x^3 = -\frac{\ln t}{2} + \frac{19}{8} + \frac{\ln 0.1}{2}$ | A1 | $10x^2 - x^3 = -\frac{\ln 2t}{2} + \frac{19}{8} + \frac{\ln 0.2}{2}$ or $10x^2 - x^3 = -\frac{\ln t}{2} + 2.5 - 0.125 - 1.15\ldots$ Allow 1.14 to 1.16 for 1.15 and allow 2.44 to 2.46 for 2.45 |
| | Obtain answer $t = \frac{1}{10} e^{2x^3 - 20x^2 + \frac{19}{4}}$ or equivalent | A1 | ISW<br>Need $t = \ldots\ldots\ldots$<br>E.g. $\frac{0.1}{e^{20x^2 - 2x^3 - \frac{19}{4}}}$, $\frac{e^{2x^3 + \frac{19}{4}}}{10e^{20x^2}}$, $\frac{1}{10} e^{\frac{19}{4}} e^{2x^3 - 20x^2}$.<br>Allow decimals, allow 2.44 to 2.46 for 2.45, e.g. $e^{2x^3 - 20x^2 + 2.45}$.<br>A0 if $e^{\ln \frac{1}{10}}$ present in final answer. |

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| 11(a) | Use of correct product rule and correct chain rule | M1 | $\frac{dy}{dx} = A\cos x\sqrt{2+\cos x} + \frac{B\sin x \sin x}{\sqrt{2+\cos x}}$ |
| | Obtain $\frac{dy}{dx} = 2\cos x\sqrt{2+\cos x} - \frac{2\sin^2 x}{2\sqrt{2+\cos x}}$ | A1 | OE |
| | Equate the derivative to zero and obtain a horizontal 3 term quadratic equation or 4 term quartic equation in $\cos a$<br>If M0 earlier then needs that expression to be such that arrive at 3 term quadratic or 4 term quartic equation in cos $x$ without further trig errors.<br>The only error in the form of the differential allowed is for $(2+\cos x)^{-\frac{1}{2}}$ to be $(2+\cos x)^{+\frac{1}{2}}$ or $(2+\cos x)^{-\frac{3}{2}}$ | *M1 | Accept in $\cos x$.<br>E.g. $3\cos^2 x + 4\cos x - 1 = 0$.<br>E.g. $3\cos^4 x + 16\cos^3 x + 18\cos^2 x - 1 = 0$. |
| | Solve for cos $a$ | DM1 | $\left(\cos a = \frac{-2+\sqrt{7}}{3} \text{ or } 0.215\right)$<br>Allow presence of other solution(s). |
| | Obtain $a = 4.93$ | A1 | Allow more accurate, e.g. 4.929… even though question states 2 dp.<br>If $x = 1.35$ leads to $x = 4.93$ award A1 BOD.<br>If $x = 1.35$ and $x = 4.93$ award A0. |

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| 11(b) | State or imply $du = -\sin x \, dx$ | B1 | OE<br>If B0, max M1M1M1. |
| | Substitute throughout for $u$ and $du$ | M1 | |
| | Obtain $-\int 2\sqrt{u}du$ | A1 | OE. Ignore limits if $-\int 2\sqrt{u}du$, but if $+\int 2\sqrt{u}du$, then must have correct limits $\int_1^3 2\sqrt{u}du$.<br>(See final M1) |
| | Integrate to obtain $ku^{\frac{3}{2}} (+C)$ | M1 | Constant of integration not required |
| | Use correct limits correctly in an expression of the form $ku^{\frac{3}{2}}$ or $k(2+\cos x)^{\frac{3}{2}}$ | M1 | 1 and 3 for $u$, or 0 and $\pi$ for $x$. |
| | Obtain $\frac{4}{3}\left(3\sqrt{3}-1\right)$ or $4\sqrt{3}-\frac{4}{3}$ or $\frac{4}{3}\sqrt{27}-\frac{4}{3}$ | A1 | OE. Allow, e.g., $\sqrt{3}^3$ for $\sqrt{27}$.<br>ISW but don't ignore e.g. multiplying throughout by 3.<br>If the answer is changed from negative to positive value at end, then A0.<br>Last M1A1 can use modulus, providing no errors seen. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | Use correct product or quotient rule | *M1 | $\frac{dy}{dx} = \left(-\frac{1}{2}\right)(2-x)e^{-\frac{1}{2}x} - e^{-\frac{1}{2}x}$<br>M1 requires at least one of derivatives correct |
| | Obtain correct derivative in any form | A1 | |
| | Equate derivative to zero and solve for $x$ | DM1 | |
| | Obtain $x = 4$ | A1 | ISW |
| | Obtain $y = -2e^{-2}$, or exact equivalent | A1 | |
| | | 5 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | Commence integration and reach $a(2-x)e^{-\frac{1}{2}x} + b\int e^{-\frac{1}{2}x}dx$ | *M1 | Condone omission of $dx$<br>$-2(2-x)e^{-\frac{1}{2}x} + 4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$ |
| | Obtain $-2(2-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}dx$ | A1 | OE |
| | Complete integration and obtain $2xe^{-\frac{1}{2}x}$ | A1 | OE |
| | Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice | DM1 | Ignore omission of zeros and allow max of 1 error |
| | Obtain answer $4e^{-1}$, or exact equivalent | A1 | ISW |

| Question | Answer | Marks | Guidance |
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| 4 | Commence integration and reach $ax\cos\frac{1}{2}x + b\int\cos\frac{1}{2}x\,dx$ | *M1 | |
| | Obtain $-2x\cos\frac{1}{2}x + 2\int\cos\frac{1}{2}x\,dx$ | A1 | OE |
| | Complete integration obtaining $-2x\cos\frac{1}{2}x + 4\sin\frac{1}{2}x$ | A1 | OE |
| | Use limits correctly, having integrated twice | DM1 | |
| | Obtain answer $2 + \frac{\sqrt{3}}{3}\pi$, or exact equivalent | A1 | |
| | | 5 | |

| Question | Answer | Marks | Guidance |
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| 10(a) | $a = 30$ and $b = 0.01$ | B1 | |
| | | 1 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | Separate variables and integrate one side | M1 | |
| | Obtain terms $-100\ln(30-0.01V)$ and $t$, or equivalent | A1 FT + A1 FT | FT *their* $a$ and $b$. |
| | Evaluate a constant, or use $t = 0$, $V = 0$ as limits, in a solution containing terms $c \ln(30-0.01V)$ and $dt$ where $cd \neq 0$ | M1 | |
| | Obtain solution $100\ln 30 - 100\ln(30-0.01V) = t$, or equivalent | A1 | |
| | Substitute $V = 1000$ and obtain answer $t = 40.5$ | A1 | |
| | | 6 | |
| 10(c) | Obtain $V = 3000(1-e^{-0.01t})$ | B1 | OE |
| | State that $V$ approaches 3000 | B1 | |
| | | 2 | |

| Question | Answer | Marks | Guidance |
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| 10(a) | State that $\frac{dV}{dt} = 50000 - 600h$ | B1 | May be seen as $\frac{dV}{dt} = 50000$ and $\frac{dV}{dt} = [-]600h$. When put together (may be in the chain rule) B1 can be awarded. |
| | [Use $V = 40000h$ to] obtain $\frac{dV}{dh} = 40000$ and use this and *their* $\frac{dV}{dt}$ in the correct chain rule to obtain $\frac{dh}{dt}$<br>or<br>[Use $V = 40000h$ to] obtain $\frac{dV}{dt} = 40000\frac{dh}{dt}$ and equate to *their* $\frac{dV}{dt}$ | M1 | $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$<br>E.g. $\frac{50000-600h}{40000} = \frac{dh}{dt}$<br>E.g. $40000\frac{dh}{dt} = 50000 - 600h$ |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | Separate variables correctly and integrate one side correctly | M1 | E.g. $\int \frac{1}{250-3h} dh = \int \frac{1}{200} dt$.<br>Integral signs may be omitted, 200 may be on opposite side. |
| | Obtain $-\frac{1}{3}\ln\lvert 250-3h\rvert = \frac{t}{200} (+C)$ | A1 | OE<br>Condone missing "$+C$" and lack of modulus signs. |
| | Use $t = 0$, $h = 50$ in an expression containing $\ln(250-3h)$ or $\ln\lvert 250-3h\rvert$ to find the constant of integration. | M1 | Or equivalent use of limits 50 and 80. |
| | Obtain $C = -\frac{1}{3}\ln 100$ | A1 | OE, e.g. $\frac{1}{3}\ln\frac{100}{250-3h} = \frac{t}{200}$, or $-\frac{200}{3}\ln\left(\frac{10}{100}\right)$.<br>With or without modulus signs on the log terms. |
| | $t = 150$ | A1 | |
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| 2 | Use correct product rule<br>$\cos 2x$ may be $1 - 2\sin^2 x$ or … | **M1** | $ae^{2x}\sin 2x + e^{2x}b\cos 2x$. Need $a$ or $b = 2$.<br>Allow M1 if only error is $e^x$ instead of $e^{2x}$ in one of terms, then maximum 1/5. |
| | Obtain correct derivative $2e^{2x}\sin 2x + 2e^{2x}\cos 2x$ | **A1** | OE, e.g. $4e^{2x}\sin x\cos x + 2e^{2x}\left(\cos^2 x - \sin^2 x\right)$. |
| | Equate derivative of the form $ae^{2x}\sin 2x + e^{2x}b\cos 2x$ to 0 and solve for $2x$ or $x$ using a correct method<br>Note may have substituted for $\sin 2x$ and/or $\cos 2x$ | **M1** | Obtain $2x = \tan^{-1}(-\ \textit{their}\ b/\textit{their}\ a)$ OE.<br>Allow one slip in rearranging.<br>Allow degrees.<br>Variety of other methods available, such as solving quadratic equation in $\sin x$ or $\tan x$ e.g. $\tan^2 x - 2\tan x - 1 = 0$ leading to $x = \tan^{-1}(1+\sqrt{2})$. |
| | Obtain $x = \frac{3}{8}\pi$ only or exact equivalent | **A1** | CWO<br>67.5° gets A0.<br>Ignore any answers outside interval $0 \leqslant x \leqslant \dfrac{\pi}{2}$. |
| | Obtain $y = \dfrac{1}{2}\sqrt{2}e^{\frac{3}{4}\pi}$ only or exact simplified equivalent | **A1** | CWO, ISW.<br>Not $\sin\left(\dfrac{3}{4}\pi e^{\frac{3}{4}\pi}\right)$.<br>Ignore any answers using $x$ outside interval $0 \leqslant x \leqslant \dfrac{\pi}{2}$. |