Question	Answer	Marks
2	Commence integration and reach $a(2-x)e^{-2x}+b\int e^{-2x}\,dx$, or equivalent	M1*
	Obtain $-\frac{1}{2}(2-x)e^{-2x}-\frac{1}{2}\int e^{-2x}\,dx$, or equivalent	A1
	Complete integration and obtain $-\frac{1}{2}(2-x)e^{-2x}+\frac{1}{4}e^{-2x}$, or equivalent	A1
	Use limits correctly, having integrated twice	DM1
	Obtain answer $\frac{1}{4}\left(3-e^{-2}\right)$, or exact equivalent	A1
		5

Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	B1	$\frac{dN}{N^{\frac{3}{2}}}=(k\cos 0.02t)\,dt$ Allow without integral signs.
	Obtain term $-\frac{2}{\sqrt{N}}$	B1	OE Ignore position of k.
	Obtain term $50\sin 0.02t$	B1	OE Ignore position of k.
	Use $t = 0$, $N = 100$ to evaluate a constant, or as limits, in a solution containing terms $\frac{a}{\sqrt{N}}$ and $b\sin 0.02t$, where $ab\neq 0$	M1	$\left[\text{e.g. } c=-0.2 \text{ or } c=\frac{-0.2}{k}\right]$
	Obtain correct solution in any form, e.g. $-\frac{2}{\sqrt{N}}=50k\sin 0.02t-0.2$	A1	OE ISW e.g. $N=\frac{1}{(25k\sin 0.02t-0.1)^2}$ $-2N^{-\frac{1}{2}}=\frac{k}{0.02}\sin 0.02t-\frac{1}{5}$ $50k\sin 0.02t=-\frac{2}{\sqrt{N}}+\frac{1}{5}$ $\frac{1}{\sqrt{N}}=-\frac{1}{2}k(50\sin 0.02t)+\frac{1}{10}$ $50\sin\left(\frac{1}{50}t\right)=-\frac{2\sqrt{N}}{kN}+\frac{20}{100k}$
8(b)	Use the substitution $N = 625$ and $t = 50$ in expression of appropriate form to evaluate k	M1	Expression must contain $a + bk\sin 0.02t$, $(\sqrt{N})^{\pm n}$, where $n = -1, 1, 3$ or 5 and a and b are constants $ab\neq 0$ or $(a + bk\sin 0.02t)^{\pm 2}$ and $(N)^{\pm n}$. Allow with k replaced by $\frac{1}{k}$, error due to $k(N^{-3/2})$ when separating variables in **8(a)**. If invert term by term when 3 terms shown then M0.
	Obtain $k = 0.00285[2148]$	A1	Must evaluate sin1. Degrees $k = 0.138$ M1 A0.
		2	

Question	Answer	Marks	Guidance
8(c)	Rearrange and obtain $N = 4(0.2 - 0.142(607)\sin 0.02t)^{-2}$ Substitution for k required	M1	Anything of the form $N = c(d - ek\sin 0.02t)^{-2}$, where c, d and e are constants $cde \neq 0$ and value of k substituted. Allow with k replaced by $1/k$, error due to $k(N^{-3/2})$ when separating variables in **8(a)**. OE ISW e.g. $N=\left(-\frac{10}{0.7125\sin 0.02t-1}\right)^2$ $N=\frac{1}{(-0.0713\sin 0.02t+0.1)^2}$ $N=\frac{100}{\left(\left(\frac{0.6}{\sin 1}\right)\sin 0.02t-1\right)^2}$ $N=\frac{1}{\left(\frac{3}{-50\sin 1}\times\sin 0.02t+\frac{1}{10}\right)^2}$ $N=\left(-\frac{0.06}{\sin 1}\sin 0.02t+0.1\right)^{-2}$ $N=\left(\frac{800}{80-57\sin 0.02t}\right)^2$ Do not need to substitute for $\sin(0.02t) = 1$, but must substitute for k.
	Accept answers between 1209 and 1215	A1	ISW Substitute $\sin 0.02t = 1$ or $t = 50\sin^{-1} 1$ or 78.5 or 25π. Answer with no working (rubric) 0/2. **SC** $N = \ldots$ not seen but correct numerical answer **B1** 1/2.

Question	Answer	Marks	Guidance
9(a)	Obtain $\frac{dV}{dt}=[\pm]\frac{k}{t}$ or $\frac{dV}{dt}=[\pm]\frac{1}{kt}$	B1	
	Obtain $\frac{dV}{dx}=20x-3x^2$	B1	
	Correct use of chain rule involving k	M1	Use $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$. Expressions for $\frac{dV}{dt}$ and $\frac{dV}{dx}$ must be seen to get M1.
	Obtain $\frac{dx}{dt}=[\pm]\frac{k}{t(20x-3x^2)}$ or equivalent,	A1	If this expression is first seen with numerical values, allow A1 when their value of k is substituted back into the general expression.
	Use $t=\frac{1}{10}$, $x=\frac{1}{2}$ and $\frac{dx}{dt}=-\frac{20}{37}$ to obtain given answer which must be stated $\frac{dx}{dt}=-\frac{20}{37}$ needed to score final A1	A1	$\frac{dx}{dt}=\frac{-1}{2t(20x-3x^2)}$ AG Need to at least see $-\frac{20}{37}=\frac{k}{\frac{1}{10}\left(10-\frac{3}{4}\right)}$ if $\frac{k}{t}$ or $-\frac{20}{37}=\frac{-k}{\frac{1}{10}\left(10-\frac{3}{4}\right)}$ if $-\frac{k}{t}$ in working for correct k. $\frac{dx}{dt}=\frac{20}{37}$ seen anywhere, then A0.

Question	Answer	Marks	Guidance
9(b)	Separate variables correctly & integrate at least one side correctly	B1	
	Obtain terms $10x^2 - x^3$	B1	May see $-10x^2 + x^3$ if negative sign moved across or e.g. $20x^2 - 2x^3$ if 2 moved across. Allow $\frac{20x^2}{2}-\frac{3x^3}{3}$.
	Obtain term $\ln t$ with 'correct' coefficient from their separation of variables, for example $a\ln t$ for $\frac{a}{t}$.	B1FT	FT sign and position of 2 from their separation but B0 if error from later manipulation.
	Use $t=\frac{1}{10}$, $x=\frac{1}{2}$ to evaluate a constant or as limits in a solution containing terms of the form x^2, x^3 and $\ln t$ (or $\ln 2t$)	M1	Allow numerical and sign errors and decimals. Allow if exponentiate before substitution, even if exponentiation done incorrectly, allow for c or e^c.
	Obtain correct answer in any form, for example $10x^2-x^3=-\frac{\ln t}{2}+\frac{19}{8}+\frac{\ln 0.1}{2}$	A1	$10x^2-x^3=-\frac{\ln 2t}{2}+\frac{19}{8}+\frac{\ln 0.2}{2}$ or $10x^2-x^3=-\frac{\ln t}{2}+2.5-0.125-1.15\ldots$ Allow 1.14 to 1.16 for 1.15 and allow 2.44 to 2.46 for 2.45

	Obtain answer $t = \frac{1}{10}e^{2x^3-20x^2+\frac{19}{4}}$ or equivalent	A1	ISW Need $t = \ldots\ldots\ldots$ E.g. $\frac{0.1}{e^{20x^2-2x^3-\frac{19}{4}}}$, $\frac{e^{2x^3+\frac{19}{4}}}{10e^{20x^2}}$, $\frac{1}{10}e^{\frac{19}{4}}e^{2x^3-20x^2}$. Allow decimals, allow 2.44 to 2.46 for 2.45, e.g. $e^{2x^3-20x^2+2.45}$. A0 if $e^{\ln\frac{1}{10}}$ present in final answer.

10(a)(i)	$(y=)10+k$	B1
(ii)	$x=\dfrac{10}{k}$ **or** $y=k$	B1
	$x=\dfrac{10}{k}$ **and** $y=k$	B1
		(3)

(b)	$kx-10+k=2k \Rightarrow x=\ldots$ **or** $-kx+10+k=2k \Rightarrow x=\ldots$	M1
	$kx-10+k=2k \Rightarrow x=\ldots$ **and** $-kx+10+k=2k \Rightarrow x=\ldots$	dM1
	x ,, $\dfrac{10-k}{k}$ or x … $\dfrac{10+k}{k}$ oe	A1
		(3)

Question	Answer	Marks	Guidance
10(a)	$a = 30$ and $b = 0.01$	**B1**	
		1	

Question	Answer	Marks	Guidance
10(b)	Separate variables and integrate one side	**M1**	
	Obtain terms $-100\ln(30-0.01V)$ and t, or equivalent	**A1 FT** **+ A1 FT**	FT *their a* and *b*.
	Evaluate a constant, or use $t=0$, $V=0$ as limits, in a solution containing terms $c\ln(30-0.01V)$ and dt where $cd \neq 0$	**M1**	
	Obtain solution $100\ln 30-100\ln(30-0.01V)=t$, or equivalent	**A1**	
	Substitute $V = 1000$ and obtain answer $t = 40.5$	**A1**	
		6	
10(c)	Obtain $V=3000\left(1-e^{-0.01t}\right)$	**B1**	OE
	State that V approaches 3000	**B1**	
		2	

Question	Answer	Marks	Guidance
5(a)	Use correct product rule	M1	$\frac{d}{dx}(x^2)\cos(3x) + x^2\frac{d}{dx}(\cos 3x)$.
	Obtain correct derivative in any form	A1	e.g. $2x\cos 3x - 3x^2\sin 3x$.
	Equate derivative to zero and obtain $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$.	A1	AG Condone $a = \frac{1}{3}\tan^{-1}\frac{2}{3a}$. Must at least reach expression $2x = 3x^2\tan(3x)$ or better before final answer to gain A1. Final answer must be in terms of a. Can work with x and switch to a at very end. Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0.
		3	

Question	Answer	Marks	Guidance
5(b)	Use the iterative process $a_{n+1} = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations	M1	Degrees 0/3.
	Obtain final answer 0.36	A1	Must be 2d.p.
	Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval $(0.355, 0.365)$	A1	Allow small errors in 4th d.p. Allow errors at start if self corrects later.
	0.5 0.4 0.3 0.2 0.1 π/6 π/12 0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989 0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439 0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649 0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567 0.3578 0.3604 0.3614 0.3576 0.3580	3	

Question	Answer	Marks	Guidance
3(a)	State $\frac{dx}{dt} = 1 + \frac{1}{t+2}$	B1	
	Use product rule	M1	
	Obtain $\frac{dy}{dt} = e^{-2t} - 2(t-1)e^{-2t}$	A1	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain correct answer in any simplified form, e.g. $\frac{(3-2t)(t+2)}{t+3}\ e^{-2t}$	A1	
		5	
3(b)	Equate derivative to zero and solve for t	M1	
	Obtain $t = \frac{3}{2}$ and obtain answer $y = \frac{1}{2}e^{-3}$, or exact equivalent	A1	

4(a)	$$\frac{49x}{x^2+x-12}+\frac{7x}{x+4}=\frac{49x+7x(x-3)}{(x+4)(x-3)}$$ or e.g. $$\frac{49x(x+4)}{(x^2+x-12)(x+4)}+\frac{7x(x^2+x-12)}{(x+4)(x^2+x-12)}=\frac{49x(x+4)+7x(x^2+x-12)}{(x+4)(x^2+x-12)}$$	M1A1
	$$\frac{49x+7x(x-3)}{(x+4)(x-3)}=\frac{49x+7x^2-21x}{(x+4)(x-3)}=\frac{7x^2+28x}{(x+4)(x-3)}$$ $$=\frac{7x\cancel{(x+4)}}{\cancel{(x+4)}(x-3)}=\frac{7x}{(x-3)}\ *$$	A1*
		(3)