6 (a) On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z-4-3\mathrm{i}| \leqslant 2$ and $\arg(z-2-\mathrm{i}) \geqslant \frac{1}{3}\pi$. [5]

(b) Calculate the greatest value of $\arg z$ for points in this region. [2]

5: Find

(i) $\int \sin^2 3x \, \mathrm{d}x$ **(2)**

(ii) $\int x(x^2+4)^{\frac{3}{2}} \, \mathrm{d}x$ **(2)**

(b) Find the gradients of the tangents to the curve when $x = 0$. [2]

4. A new mobile phone is released for sale.

The total sales N of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$$

where A is a constant and t is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

(a) state the value of A. **(1)**

Given that when $t = T$ the total sales of the phone was 100000

(b) find, according to the model, the value of T. Give your answer to 2 decimal places. **(3)**

(c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*) **(2)**

The total sales of the mobile phone is expected to reach 150000

Using this information,

(d) give a reason why the given equation is not suitable for modelling the total sales of the phone. **(1)**

7: A continuous curve has equation

$$y = e^{-x^2}\sin 3x \qquad 0 \leqslant x \leqslant \frac{\pi}{3}$$

The curve has a stationary point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3}\arctan\left(\frac{3}{2x}\right)$$

(4)

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [4]

(c) State the set of values of x for which the expansion in part **(b)** is valid. [1]

4 The curve $y = e^{-4x}\tan x$ has two stationary points in the interval $0 \leqslant x < \frac{1}{2}\pi$.

(a) Obtain an expression for $\frac{dy}{dx}$ and show it can be written in the form $\sec^2 x(a + b\sin 2x)e^{-4x}$, where a and b are constants. [4]

(b) Hence find the exact x-coordinates of the two stationary points. [3]

6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ°C, of a computer processor, t minutes after the computer is switched off, is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the processor was 75°C when the computer was switched off,

(a) find the value of A.

(2)

Given also that it takes 5 minutes for the temperature of the processor to decrease from 75°C to 25°C,

(b) find the value of k, giving your answer to 3 significant figures.

(3)

At time T minutes, the temperature of the processor is decreasing at a rate of 9°C per minute.

(c) Find the value of T according to the model, giving your answer to 2 decimal places.

(3)

3 **(a)** Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x. [2]

(b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places. [4]

10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$x = \sin^2 4y \qquad 0 \leqslant y \leqslant \frac{\pi}{8} \qquad 0 \leqslant x \leqslant 1$$

The point P with x coordinate $\frac{1}{4}$ lies on C

(a) Find the exact y coordinate of P

(2)

(b) Find $\frac{dx}{dy}$

(2)

(c) Hence show that $\frac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \frac{1}{\sqrt{q + r(x+s)^2}}$$

where q, r and s are constants to be found.

(3)

Using the answer to part (c),

(d) (i) state the x coordinate of the point where the value of $\frac{dy}{dx}$ is a minimum,

(ii) state the value of $\frac{dy}{dx}$ at this point.

(2)

DO NOT WRITE IN THIS AREA

(b) Find the exact area of the shaded region shown in the diagram, bounded by the curve and the x-axis. [5]