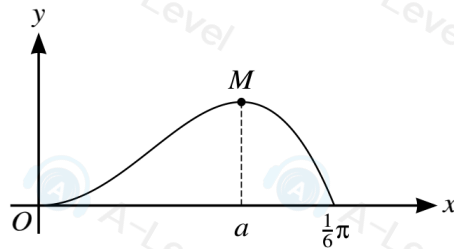


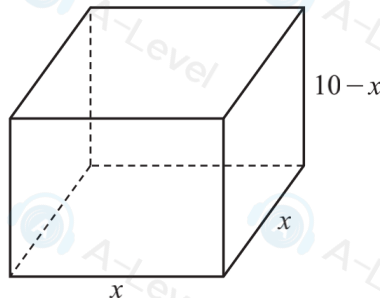
5



The diagram shows the part of the curve  $y = x^2 \cos 3x$  for  $0 \leq x \leq \frac{1}{6}\pi$ , and its maximum point  $M$ , where  $x = a$ .

- (a) Show that  $a$  satisfies the equation  $a = \frac{1}{3} \tan^{-1} \left( \frac{2}{3a} \right)$ . [3]
- (b) Use an iterative formula based on the equation in (a) to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9



A container in the shape of a cuboid has a square base of side  $x$  and a height of  $(10 - x)$ . It is given that  $x$  varies with time,  $t$ , where  $t > 0$ . The container decreases in volume at a rate which is inversely proportional to  $t$ .

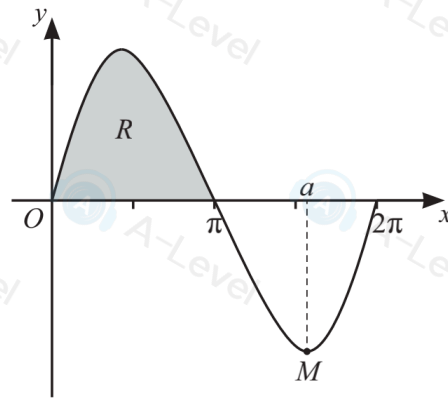
When  $t = \frac{1}{10}$ ,  $x = \frac{1}{2}$  and the rate of decrease of  $x$  is  $\frac{20}{37}$ .

- (a) Show that  $x$  and  $t$  satisfy the differential equation

$$\frac{dx}{dt} = \frac{-1}{2t(20x - 3x^2)}. \quad [5]$$

- (b) Solve the differential equation, obtaining an expression for  $t$  in terms of  $x$ . [6]

11

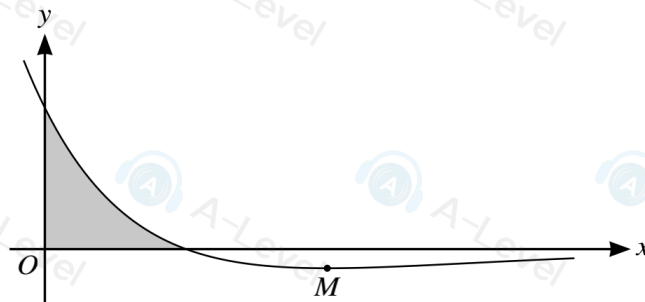


The diagram shows the curve  $y = 2 \sin x \sqrt{2 + \cos x}$ , for  $0 \leq x \leq 2\pi$ , and its minimum point  $M$ , where  $x = a$ .

(a) Find the value of  $a$  correct to 2 decimal places. [5]

(b) Use the substitution  $u = 2 + \cos x$  to find the exact area of the shaded region  $R$ . [6]

10



The diagram shows the curve  $y = (2 - x)e^{-\frac{1}{2}x}$ , and its minimum point  $M$ .

(a) Find the exact coordinates of  $M$ . [5]

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of  $e$ . [5]

4 Find the exact value of  $\int_{\frac{1}{3}\pi}^{\pi} x \sin \frac{1}{2}x \, dx$ . [5]

- 10 A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time  $t$  minutes after filling begins the volume of water in the pool is  $V$  litres. The pool has a small leak and loses water at a rate of  $0.01V$  litres per minute.

The differential equation satisfied by  $V$  and  $t$  is of the form  $\frac{dV}{dt} = a - bV$ .

- (a) Write down the values of the constants  $a$  and  $b$ . [1]

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- (b) Solve the differential equation and find the value of  $t$  when  $V = 1000$ . [6]

- (c) Obtain an expression for  $V$  in terms of  $t$  and hence state what happens to  $V$  as  $t$  becomes large. [2]

- 10 A water tank is in the shape of a cuboid with base area  $40\,000\text{ cm}^2$ . At time  $t$  minutes the depth of water in the tank is  $h$  cm. Water is pumped into the tank at a rate of  $50\,000\text{ cm}^3$  per minute. Water is leaking out of the tank through a hole in the bottom at a rate of  $600h\text{ cm}^3$  per minute.

- (a) Show that  $200\frac{dh}{dt} = 250 - 3h$ . [3]

- (b) It is given that when  $t = 0$ ,  $h = 50$ .

Find the time taken for the depth of water in the tank to reach 80 cm. Give your answer correct to 2 significant figures. [5]

- 2 Find the exact coordinates of the stationary point of the curve  $y = e^{2x} \sin 2x$  for  $0 \leq x \leq \frac{1}{2}\pi$ . [5]