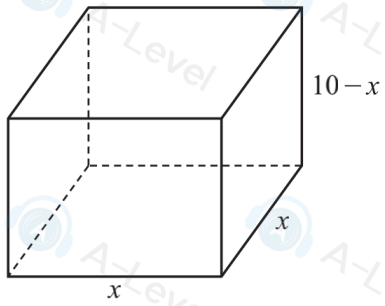
2 Find the exact value of $\int_0^1 (2-x)e^{-2x}\,dx$. [5]

8 At time t days after the start of observations, the number of insects in a population is N. The variation in the number of insects is modelled by a differential equation of the form $\frac{dN}{dt} = kN^{\frac{3}{2}}\cos 0.02t$, where k is a constant and N is a continuous variable. It is given that when $t = 0$, $N = 100$.

(a) Solve the differential equation, obtaining a relation between N, k and t. [5]

(b) Given also that $N = 625$ when $t = 50$, find the value of k. [2]

9



A container in the shape of a cuboid has a square base of side x and a height of $(10-x)$. It is given that x varies with time, t, where $t > 0$. The container decreases in volume at a rate which is inversely proportional to t.

When $t = \frac{1}{10}$, $x = \frac{1}{2}$ and the rate of decrease of x is $\frac{20}{37}$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{-1}{2t(20x-3x^2)}.$$ [5]

(b) Solve the differential equation, obtaining an expression for t in terms of x. [6]

10:

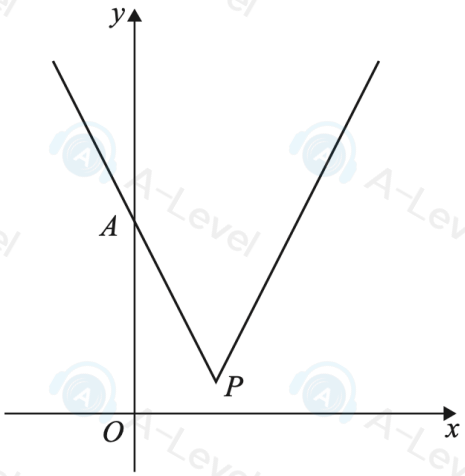


Figure 3

Figure 3 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 10| + k \qquad x \in \mathbb{R}$$

and k is a positive constant.

The graph

- cuts the y-axis at the point A
- has a vertex at the point P

(a) Find, in simplest form in terms of k,

(i) the y coordinate of A

(ii) the coordinates of P

(3)

(b) Find, in terms of k, the range of values of x which satisfy

$$|kx - 10| + k \geqslant 2k$$

(3)

Given that the line with equation $y = 3x + 1$ intersects the graph of $y = f(x)$ at 2 distinct points,

(c) find the range of values of k.

(4)

10 A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\frac{\mathrm{d}V}{\mathrm{d}t} = a - bV$.

(a) Write down the values of the constants a and b. [1]

.....

.....

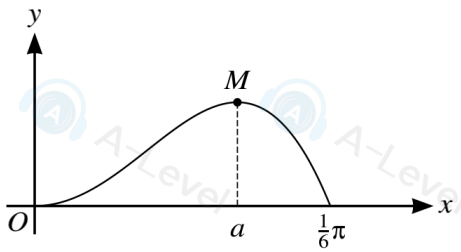
.....

.....

(b) Solve the differential equation and find the value of t when $V = 1000$. [6]

(c) Obtain an expression for V in terms of t and hence state what happens to V as t becomes large. [2]

5



The diagram shows the part of the curve $y = x^2 \cos 3x$ for $0 \leqslant x \leqslant \frac{1}{6}\pi$, and its maximum point M, where $x = a$.

(a) Show that a satisfies the equation $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$. [3]

(b) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3 The parametric equations of a curve are

$$x = t + \ln(t + 2), \qquad y = (t - 1)\mathrm{e}^{-2t},$$

where $t > -2$.

(a) Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, simplifying your answer. [5]

(b) Find the exact y-coordinate of the stationary point of the curve. [2]

4: The function f is defined by

$$f(x) = \frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} \qquad x > 3$$

(a) Show that

$$f(x) = \frac{7x}{x - 3} \qquad x > 3$$

(3)

(b) Hence find $f'(x)$ giving your answer in simplest form.

(2)

(b) $z = 3e^{\frac{1}{4}\pi i}$ is a root of the equation $z^2 + bz + c = 0$, where b and c are real.

State the other root and hence find the values of b and c. [3]