

9 Let $f(x) = \frac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

9.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

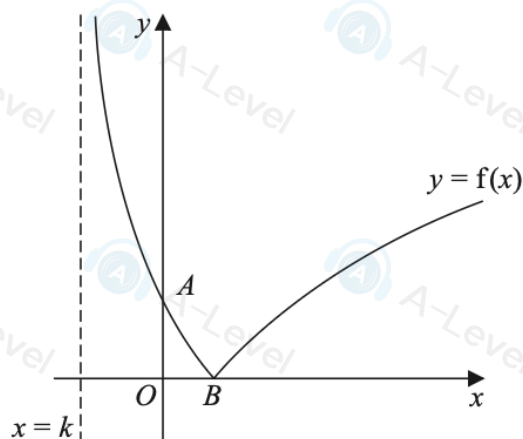


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x + 1)| \quad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y -axis at point A
- meets the x -axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A

(ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4\ln(x + 1)| > 3$$

(5)

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . [5]

(c) State the set of values of x for which the expansion in (b) is valid. Give your answer in an exact form. [1]

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10 Let $f(x) = \frac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 .

[5]

8.

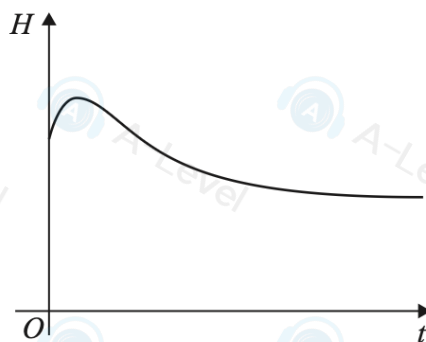


Figure 4

The heart rate of a horse is being monitored.

The heart rate H , measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20e^{-0.9t}$$

where t minutes is the time after monitoring began.

Figure 4 is a sketch of H against t .

Use the equation of the model to answer parts (a) to (e).

(a) State the initial heart rate of the horse.

(1)

In the long term, the heart rate of the horse approaches L bpm.

(b) State the value of L .

(1)

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The heart rate of the horse reaches its maximum value after T minutes.

(c) Find the value of T , giving your answer to 3 decimal places.

(Solutions based entirely on calculator technology are not acceptable.)

(5)

The heart rate of the horse is 37 bpm after M minutes.

(d) Show that M is a solution of the equation

$$t = 5 \ln \left(\frac{8}{1 + 4e^{-0.9t}} \right)$$

(2)

Using the iteration formula

$$t_{n+1} = 5 \ln \left(\frac{8}{1 + 4e^{-0.9t_n}} \right) \quad \text{with} \quad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of t_2

(ii) find, to 4 decimal places, the value of M

(3)

7: A continuous curve has equation

$$y = e^{-x^2} \sin 3x \quad 0 \leq x \leq \frac{\pi}{3}$$

The curve has a stationary point at the point P .

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3} \arctan \left(\frac{3}{2x} \right)$$

(4)

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [4]

(c) State the set of values of x for which the expansion in part (b) is valid. [1]

1 Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

7 Let $f(x) = \frac{5x^2 + 8x - 3}{(x - 2)(2x^2 + 3)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

- 2 Expand $\sqrt{\frac{1+2x}{1-2x}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [5]