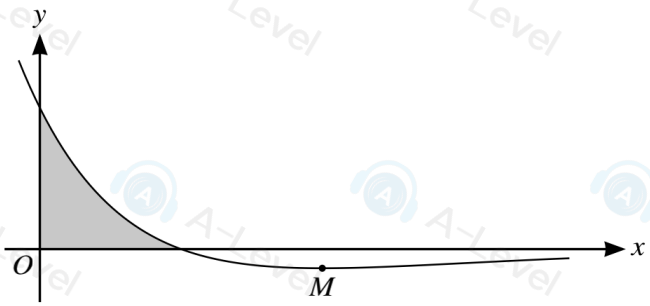
10



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e. [5]

4 The parametric equations of a curve are

$$x = 2t - \tan t, \qquad y = \ln(\sin 2t),$$

for $0 < t < \frac{1}{2}\pi$.

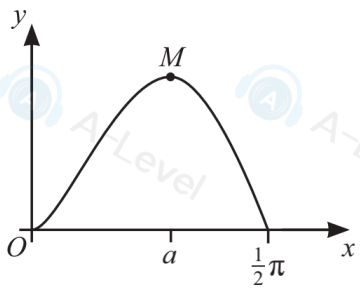
Show that $\dfrac{dy}{dx} = \cot t$. [5]

4 The curve $y = e^{-4x}\tan x$ has two stationary points in the interval $0 \leqslant x < \frac{1}{2}\pi$.

(a) Obtain an expression for $\dfrac{dy}{dx}$ and show it can be written in the form $\sec^2 x(a + b\sin 2x)e^{-4x}$, where a and b are constants. [4]

(b) Hence find the exact x-coordinates of the two stationary points. [3]

11



The diagram shows the curve $y = \sqrt{x}\sin 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve has a maximum point at M, where $x = a$.

(a) Show that $\tan 2a = -4a$ [4]

(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\left(\pi - \tan^{-1}\left(4x_n\right)\right)$$

converges, then it converges to a. [2]

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(d) Use the iterative formula in part **(c)** to calculate a correct to 4 decimal places. Give the result of each iteration to 6 decimal places. [3]

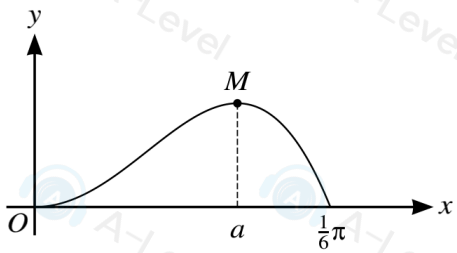
(b) Show by calculation that $0.9 < a < 0.95$. [2]

4 The parametric equations of a curve are

$$x = \frac{\cos\theta}{2 - \sin\theta}, \qquad y = \theta + 2\cos\theta.$$

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (2 - \sin\theta)^2$. [5]

5



The diagram shows the part of the curve $y = x^2 \cos 3x$ for $0 \leqslant x \leqslant \frac{1}{6}\pi$, and its maximum point M, where $x = a$.

(a) Show that a satisfies the equation $a = \frac{1}{3}\tan^{-1}\left(\dfrac{2}{3a}\right)$. [3]

(b) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The equation of a curve is $\ln(x + y) = x - 2y$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x + y - 1}{2(x + y) + 1}$. [4]

(b) Find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]