3:

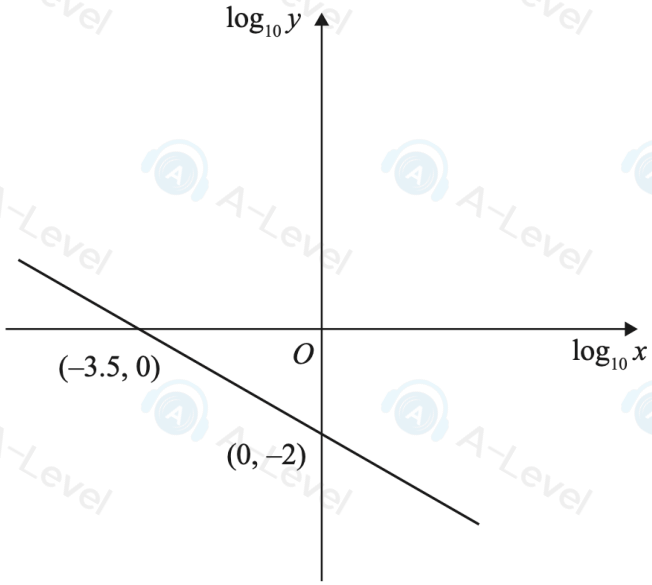


Figure 1

Figure 1 shows a linear relationship between $\log_{10} y$ and $\log_{10} x$

The line passes through the points (–3.5, 0) and (0, –2) as shown.

(a) Find an equation linking $\log_{10} y$ with $\log_{10} x$

(2)

(b) Hence, or otherwise, express y in the form px^q where p and q are rational constants.

(3)

7 **(a)** Use the substitution $u = \cos x$ to show that

$$\int_0^{\pi} \sin 2x \, e^{2\cos x} \, dx = \int_{-1}^{1} 2ue^{2u} \, du. \qquad [4]$$

(b) Hence find the exact value of $\int_0^{\pi} \sin 2x \, e^{2\cos x} \, dx$. [4]

8 At time t days after the start of observations, the number of insects in a population is N. The variation in the number of insects is modelled by a differential equation of the form $\frac{dN}{dt} = kN^{\frac{3}{2}} \cos 0.02t$, where k is a constant and N is a continuous variable. It is given that when $t = 0$, $N = 100$.

(a) Solve the differential equation, obtaining a relation between N, k and t. [5]

(b) Given also that $N = 625$ when $t = 50$, find the value of k. [2]

9 Let $f(x) = \frac{1}{(9-x)\sqrt{x}}$.

(a) Find the x-coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

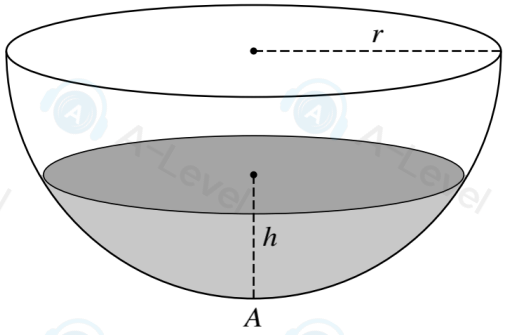
(b) Using the substitution $u = \sqrt{x}$, show that $\int_0^4 f(x) \, dx = \frac{1}{3} \ln 5$. [6]

11 Let $\mathrm{f}(x) = \dfrac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Find the exact value of $\displaystyle\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$, simplifying your answer. [5]

10



A tank containing water is in the form of a hemisphere. The axis is vertical, the lowest point is A and the radius is r, as shown in the diagram. The depth of water at time t is h. At time $t = 0$ the tank is full and the depth of the water is r. At this instant a tap at A is opened and water begins to flow out at a rate proportional to $\sqrt{h}$. The tank becomes empty at time $t = 14$.

The volume of water in the tank is V when the depth is h. It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

(a) Show that h and t satisfy a differential equation of the form

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

where B is a positive constant. [4]

(b) Solve the differential equation and obtain an expression for t in terms of h and r. [8]