11 The complex number z is defined by $z = \dfrac{5a - 2\mathrm{i}}{3 + a\mathrm{i}}$, where a is an integer. It is given that $\arg z = -\frac{1}{4}\pi$.

(a) Find the value of a and hence express z in the form $x + \mathrm{i}y$, where x and y are real. [6]

(b) Express z^3 in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the simplified exact values of r and θ. [3]

6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ°C, of a computer processor, t minutes after the computer is switched off, is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the processor was 75°C when the computer was switched off,

(a) find the value of A.

(2)

Given also that it takes 5 minutes for the temperature of the processor to decrease from 75°C to 25°C,

(b) find the value of k, giving your answer to 3 significant figures.

(3)

At time T minutes, the temperature of the processor is decreasing at a rate of 9°C per minute.

(c) Find the value of T according to the model, giving your answer to 2 decimal places.

(3)

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10 **(a)** Verify that $-1 + \sqrt{2}\mathrm{i}$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

(b) Find the other roots of this equation. [7]

2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geqslant 2$ and $|z - 1 + \mathrm{i}| \leqslant 1$. [4]

4. A new mobile phone is released for sale.

The total sales N of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$$

where A is a constant and t is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

(a) state the value of A.

(1)

Given that when $t = T$ the total sales of the phone was 100 000

(b) find, according to the model, the value of T. Give your answer to 2 decimal places.

(3)

(c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

The total sales of the mobile phone is expected to reach 150 000

Using this information,

(d) give a reason why the given equation is not suitable for modelling the total sales of the phone.

(1)

6 **(a)** On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z-4-3\mathrm{i}| \leqslant 2$ and $\arg(z-2-\mathrm{i}) \geqslant \frac{1}{3}\pi$. [5]

(b) Calculate the greatest value of $\arg z$ for points in this region. [2]

5 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z+2| \leqslant 2$ and $\mathrm{Im}\, z \geqslant 1$. [4]

(b) Find the greatest value of $\arg z$ for points in the shaded region. [2]

3 The square roots of $24-7\mathrm{i}$ can be expressed in the Cartesian form $x+\mathrm{i}y$, where x and y are real and exact.

By first forming a quartic equation in x or y, find the square roots of $24-7\mathrm{i}$ in exact Cartesian form. [5]

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