

Question	Answer	Marks	Guidance
6(a)	$p + q + 0.65 = 1$	B1	Sum of probabilities = 1.
	$p + 2q + 0.15 = 0.55$	B1	Use given information.
	Solve 2 linear equations	M1	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$p = 0.3, \frac{3}{10}, q = 0.05, \frac{1}{20}$	A1	CAO, both WWW If M0 with correct answers SC B1 .
		4	
6(b)	$\text{Var}(X) = \text{their } 0.3 + 4 \times \text{their } 0.05 + 9 \times 0.05 - 0.55^2$	M1	Appropriate variance formula including $(E(X))^2$, accept unsimplified.
	$0.6475 \left[\frac{259}{400} \right]$	A1	CAO (must be exact).
		2	
6(c)	$1 - P(0, 1, 2) = 1 - ({}^{12}C_0 0.3^0 0.7^{12} + {}^{12}C_1 0.3^1 0.7^{11} + {}^{12}C_2 0.3^2 0.7^{10})$	M1	One correct term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0 < x < 12$, $0 < p < 1$.
	$1 - (0.01384 + 0.07118 + 0.16779)$	A1FT	Correct unsimplified expression, or better in final answer. Unsimplified expression must be seen to FT <i>their</i> p from 6(a) or correct.
	0.747	A1	
		3	
6(d)	$(0.95)^8 \times 0.05 = 0.0332$ or $0.95^8 - 0.95^9 = 0.0332$	B1	Evaluated.
		1	

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Question	Answer	Marks	Guidance
2	$p + p + 0.1 + q + q = 1$	B1	Sum of probabilities = 1
	$0.1 + 2q = 3(2p)$	B1	Use given information
	Attempt to solve two correct equations in p and q	M1	Either use of Substitution method to form a single equation in either p or q and finding values for both unknowns. Or use of Elimination method by writing both equations in same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
	$p = \frac{1}{8}$ or 0.125 and $q = \frac{13}{40}$ or 0.325	A1	CAO, both WWW
		4	

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Question	Answer						Marks	Guidance
4(a)	x	0	1	2	3	4	B1	Table with correct x values and one correct probability.
	$P(X = x)$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{4}{16}$	B1	2 more probabilities correct linked to the correct x value, need not be in table, accept un-simplified.
		0.1875	0.0625	0.1875	0.3125	0.25	B1	5 correct probabilities linked with correct outcomes.
	Accept	0.188	0.0625	0.188	0.313	0.25		SCB1 5 non-zero probabilities (not all 1/5) summing to 1 placed in a probability distribution table with correct x values if only one of the first two B marks are scored.
							3	

Question	Answer	Marks	Guidance
4(b)	$[E(X) =] [0] + 1 \times \frac{1}{16} + 2 \times \frac{3}{16} + 3 \times \frac{5}{16} + 4 \times \frac{4}{16}$ $= \frac{[0] + 1 + 6 + 15 + 16}{16}$ $= \frac{38}{16} = \frac{19}{8}$	M1FT	Accept un-simplified or bold expression with probabilities ($0 < p < 1$) that add to 1. May be calculated in variance. FT <i>their</i> table with 4 or 5 probabilities adding to 1.
	$\text{Var}(X) = \frac{1 + 3 \times 2^2 + 5 \times 3^2 + 4 \times 4^2}{16} - \left(\frac{19}{8}\right)^2$ $= \frac{1 + 3 \times 4 + 5 \times 9 + 4 \times 16}{16} - \frac{361}{64}$ $= \frac{122}{16} - \left(\frac{19}{8}\right)^2$	M1FT	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 4 or more probabilities ($0 < p < 1$) which need not sum to 1 or with an expression no more evaluated than shown.
	$= \frac{127}{64}, 1\frac{63}{64}, 1.98$	A1	Accept 1.984375 to 3 or more SF. SCB1 for 1.98 if M0 M1 scored.
		3	

Question	Answer	Marks	Guidance
4(c)	Method 1		
	$1 - \left(\frac{3}{4}\right)^7$	M1	$1 - \left(\frac{3}{4}\right)^n$ with $n = 6, 7, 8$.
	$= 0.867$	A1	$= 0.8665$ to 4SF.
	Method 2		
	$[P(X < 8) =] \frac{1}{4} + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) +$ $\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^6\left(\frac{1}{4}\right)$	M1	Condone extra term $\left(\frac{3}{4}\right)^7\left(\frac{1}{4}\right)$ or the term $\left(\frac{3}{4}\right)^6\left(\frac{1}{4}\right)$ missing.
	$= 0.867$	A1	
		2	

Question	Answer	Marks	Guidance																				
3(a)	$[P(X = 4) = 3P(X = 2)]$ $4k(4 + a) = 3 \times 2k(2 + a)$ $16k + 4ak = 12k + 6ak$	M1	Using $P(X = 4) = 3P(X = 2)$ to form an equation in a and k .																				
	$a = 2$	A1	If M0 scored, SC B1 for $a = 2$ www.																				
	$3k + 8k + 15k + 24k = 1$	M1	Using sum of probabilities = 1 to form an equation in k : $k(1 + a) + 2k(2 + a) + 3k(3 + a) + 4k(4 + a) = 1$.																				
	$k = \frac{1}{50}$	A1	If M0 scored, SC B1 for $k = \frac{1}{50}$ www.																				
		4																					
3(b)	<table border="1"><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X)$</td><td>$\frac{3}{50}, 0.06$</td><td>$\frac{8}{50}, 0.16$</td><td>$\frac{15}{50}, 0.3$</td><td>$\frac{24}{50}, 0.48$</td></tr></table>	X	1	2	3	4	$P(X)$	$\frac{3}{50}, 0.06$	$\frac{8}{50}, 0.16$	$\frac{15}{50}, 0.3$	$\frac{24}{50}, 0.48$	B1 FT	<table border="1"><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X)$</td><td>$k(1 + a)$</td><td>$2k(2 + a)$</td><td>$3k(3 + a)$</td><td>$4k(4 + a)$</td></tr></table> $0 < p < 1$ for all outcomes, must be numerical.	X	1	2	3	4	$P(X)$	$k(1 + a)$	$2k(2 + a)$	$3k(3 + a)$	$4k(4 + a)$
	X	1	2	3	4																		
$P(X)$	$\frac{3}{50}, 0.06$	$\frac{8}{50}, 0.16$	$\frac{15}{50}, 0.3$	$\frac{24}{50}, 0.48$																			
X	1	2	3	4																			
$P(X)$	$k(1 + a)$	$2k(2 + a)$	$3k(3 + a)$	$4k(4 + a)$																			
		1																					
3(c)	$\text{Var}(X) = \frac{3}{50} \times 1 + \frac{8}{50} \times 2^2 + \frac{15}{50} \times 3^2 + \frac{24}{50} \times 4^2 - 3.2^2$	M1	Correct formula for variance method from their probability distribution table, $0 \leq \text{their } P(x) \leq 1$. Accept $\frac{3 + 32 + 135 + 384}{50} - \frac{256}{25}$.																				
	$[= 11.08 - 3.2^2] = 0.84[0], \frac{21}{25}$	A1	If M0 score SC B1 for 0.84 www.																				
		2																					

Question	Answer	Marks	Guidance																								
2(a)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>6</td></tr><tr><td>$P(X=x)$</td><td>$\frac{6}{36}$</td><td>$\frac{12}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{6}{36}$</td></tr><tr><td></td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td></tr><tr><td></td><td>0.167</td><td>0.333</td><td>0.167</td><td>0.167</td><td>0.167</td></tr></table>	x	1	2	3	4	6	$P(X=x)$	$\frac{6}{36}$	$\frac{12}{36}$	$\frac{6}{36}$	$\frac{6}{36}$	$\frac{6}{36}$		$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		0.167	0.333	0.167	0.167	0.167	B1	Table with correct x values and at least one correct probability linked with the correct x -value. Values need not be in order, lines may not be drawn, may be vertical, x and $P(X)$ may be omitted. Condone any additional x values if probability stated as 0.
	x	1	2	3	4	6																					
	$P(X=x)$	$\frac{6}{36}$	$\frac{12}{36}$	$\frac{6}{36}$	$\frac{6}{36}$	$\frac{6}{36}$																					
		$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$																					
	0.167	0.333	0.167	0.167	0.167																						
B1	4 correct probabilities linked with the correct x -values, need not be in table, accept unsimplified.																										
B1	5 correct probabilities linked with correct x -values, may not be in table. Decimals correct to at least 3 SF. SC B1 4 or 5 probabilities summing to 1 placed in a probability distribution table with 4 or 5 x -values between 1 and 6 inclusive.																										
		3																									
2(b)	$[E(X) = \frac{1}{36}(6+24+18+24+36) =] 3$	B1 FT	FT <i>their</i> table with 4 or 5 probabilities ($0 < p < 1$) summing to 1.																								
		1																									

Question	Answer	Marks	Guidance																									
1(a)	$2k+6k+12k+20k=1, \left[k = \frac{1}{40} \right]$	M1	Using sum of probabilities = 1 to form an equation in k . Accept $1 \times 2 \times k + 2 \times 3 \times k + 3 \times 4 \times k + 4 \times 5 \times k = 1$.																									
	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X)$</td><td>$\frac{2}{40}$</td><td>$\frac{6}{40}$</td><td>$\frac{12}{40}$</td><td>$\frac{20}{40}$</td></tr><tr><td></td><td>0.05</td><td>0.15</td><td>0.3</td><td>0.5</td></tr></table>	X	1	2	3	4	$P(X)$	$\frac{2}{40}$	$\frac{6}{40}$	$\frac{12}{40}$	$\frac{20}{40}$		0.05	0.15	0.3	0.5	M1	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X)$</td><td>$2k$</td><td>$6k$</td><td>$12k$</td><td>$20k$</td></tr></table> Two correctly linked, accurate probabilities. May be in terms of k . May not be in a table.	X	1	2	3	4	$P(X)$	$2k$	$6k$	$12k$	$20k$
	X	1	2	3	4																							
	$P(X)$	$\frac{2}{40}$	$\frac{6}{40}$	$\frac{12}{40}$	$\frac{20}{40}$																							
	0.05	0.15	0.3	0.5																								
X	1	2	3	4																								
$P(X)$	$2k$	$6k$	$12k$	$20k$																								
	A1	Table with correct X values and correct probabilities.																										
	3																											
1(b)	$[E(X) =] [E(X) = \frac{1 \times 2 + 2 \times 6 + 3 \times 12 + 4 \times 20}{40}] \frac{2 + 12 + 36 + 80}{40}$	M1	$[E(X) = 1 \times 2k + 2 \times 6k + 3 \times 12k + 4 \times 20k = 130k]$ Accept unsimplified expression. May be calculated in variance. FT <i>their</i> table with 3 or more probabilities summing to 1 ($0 < p < 1$). If there are outcomes in the table without probabilities, condone and treat as $p = 0$.																									
	$\left[\text{Var}(X) = \frac{1^2 \times 2 + 2^2 \times 6 + 3^2 \times 12 + 4^2 \times 20}{40} - (\text{their } E(X))^2 = \right]$ $\frac{1 \times 2 + 4 \times 6 + 9 \times 12 + 16 \times 20}{40} - \left(\text{their } \frac{13}{4} \right)^2$ $\left[\frac{2 + 24 + 108 + 320}{40} - \left(\text{their } \frac{13}{4} \right)^2 \right]$	M1	$[\text{Var}(X) = 1^2 \times 2k + 2^2 \times 6k + 3^2 \times 12k + 4^2 \times 20k - (130k)^2]$ Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the highlighted calculation (or less simplified) seen, Note: if table is correct, $\frac{454}{40} \left(\text{or } \frac{227}{20} \text{ or any calculation} \right) - (\text{their } E(X))^2$ implies M1.																									
	$E(X) = \frac{13}{4}, 3\frac{1}{4}, 3.05 \quad \text{Var}(X) = \frac{63}{80}, 0.7875$	A1	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in variance. Condone E, V, μ, σ etc. If A0 earned, SC B1 for identified correct final solutions.																									
		3																										

Question	Answer	Marks														
4(a)	<table border="1"><tr><td>-1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td><td>2</td></tr><tr><td>2</td><td>3</td><td>3</td><td>4</td></tr></table>	-1	0	0	1	0	1	1	2	2	3	3	4			
	-1	0	0	1												
	0	1	1	2												
	2	3	3	4												
	<table border="1"><tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Probability</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$</td></tr></table>	x	-1	0	1	2	3	4	Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	
	x	-1	0	1	2	3	4									
Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$										
Probability distribution table with correct scores with at least one probability	B1															
At least 4 probabilities correct	B1															
All probabilities correct	B1															
	3															
4(b)	$E(X) = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$	B1														
	$Var(X) = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$	M1														
	$\frac{37}{18} (= 2.06)$	A1														
		3														

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6(a)	Scenarios: HHT: $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{5} = \frac{4}{45}$ HTH: $\frac{2}{3} \times \frac{1}{3} \times \frac{4}{5} = \frac{8}{45}$ THH: $\frac{1}{3} \times \frac{2}{3} \times \frac{4}{5} = \frac{8}{45}$	M1	One 3 factor probability with 3, 3, 5 as denominators										
		M1	3 factor probabilities for 2 or 3 correct scenarios added, no incorrect scenarios										
	Total = $\frac{20}{45} = \frac{4}{9}$	A1	AG, Total of 3 products with clear context										
		3											
6(b)	<table border="1"><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob.</td><td>$\frac{1}{45}$</td><td>$\frac{8}{45}$</td><td>$\frac{20}{45}$</td><td>$\frac{16}{45}$</td></tr></table>	x	0	1	2	3	Prob.	$\frac{1}{45}$	$\frac{8}{45}$	$\frac{20}{45}$	$\frac{16}{45}$	B1	Probability distribution table with correct outcomes with at least one probability, allow extra outcome values if probability of zero stated'
	x	0	1	2	3								
	Prob.	$\frac{1}{45}$	$\frac{8}{45}$	$\frac{20}{45}$	$\frac{16}{45}$								
		B1	2 of P(0), P(1) and P(3) correct										
	B1 FT	3 or 4 probabilities sum to 1 with P(2) correct											
	3												
6(c)	$\text{Var}(X) = \frac{0^2 \times 1 + 1^2 \times 8 + 2^2 \times 20 + 3^2 \times 16}{45} - \left(\frac{32}{15}\right)^2$ $= \frac{8}{45} + \frac{80}{45} + \frac{144}{45} - \left(\frac{32}{15}\right)^2$	M1	Substitute <i>their</i> attempts at scores in correct variance formula, must have '- mean ² ' (FT if calculated) (condone probs not summing to 1); must be at least 2 non-zero values										
	$\frac{136}{225}$ or 0.604	A1											
		2											