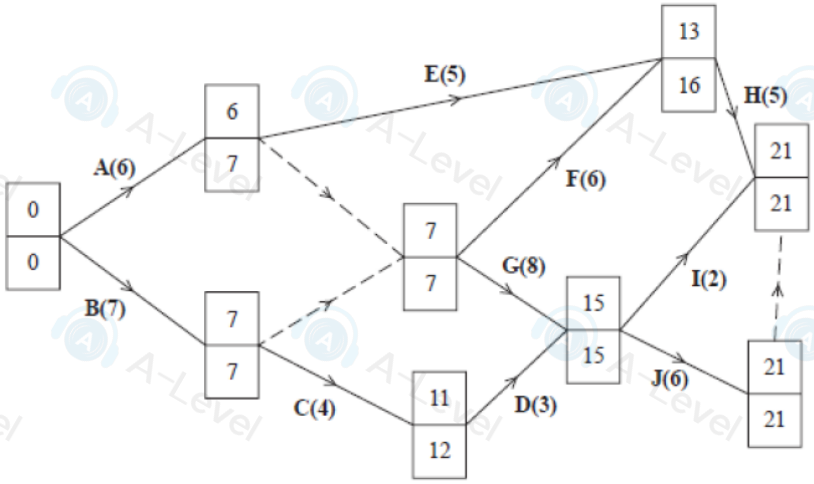
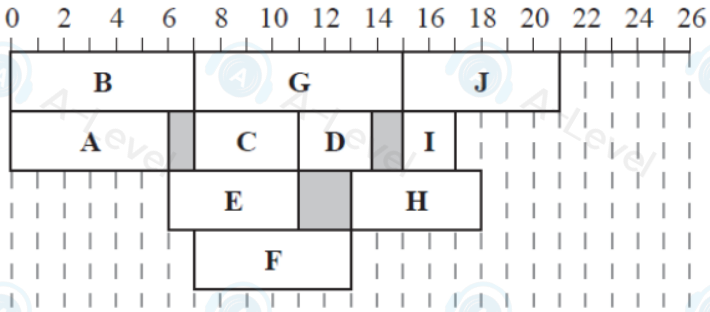
<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>5. (a)</td><td>
<table>
<tr><th>Activity</th><th>Immediately preceding activities</th><th>Activity</th><th>Immediately preceding activities</th></tr>
<tr><td>A</td><td>-</td><td>F</td><td>A, B</td></tr>
<tr><td>B</td><td>-</td><td>G</td><td>A, B</td></tr>
<tr><td>C</td><td>B</td><td>H</td><td>E, F</td></tr>
<tr><td>D</td><td>C</td><td>I</td><td>D, G</td></tr>
<tr><td>E</td><td>A</td><td>J</td><td>D, G</td></tr>
</table>
</td><td>B2, 1, 0 (2)</td></tr>
<tr><td>(b)</td><td>
</td><td>M1
A1
A1 (3)</td></tr>
<tr><td>(c)</td><td>Minimum project completion time is 21 (hours)
Critical activities: B, G, J</td><td>B1ft
B1 (2)</td></tr>
<tr><td>(d)</td><td>E could be delayed by 16 – 5 – 6 = 5 (hours)</td><td>B1 (1)</td></tr>
<tr><td>(e)</td><td>Lower bound = $\frac{52}{21} = 2.476...$ so 3 workers required</td><td>B1 (1)</td></tr>
<tr><td>(f)</td><td>e.g.

</td><td>M1
A1
A1 (3)</td></tr>
<tr><td>(g)</td><td>Activities A, E and H are now critical
The minimum project completion time is now 22 (hours)</td><td>B1
B1 (2)</td></tr>
<tr><td></td><td></td><td>14 marks</td></tr>
</table>

Question Number	Scheme	Marks

Notes for Question 5

a1B1: **Any** 7 of the 10 rows correct (allow A and B blank)
a2B1: CAO (allow A and B blank)

b1M1: All top boxes and all bottom boxes completed. Values generally increasing left to right (for top boxes) and values generally decreasing from right to left (for bottom boxes). Condone missing 0 or 21 for M only (in bottom boxes). Condone one rogue value in top boxes and one rogue value in bottom boxes. For a rogue in the top boxes if values do not increase in the direction of the arrows then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value (with a similar definition for bottom boxes but in reverse)

b1A1: CAO (top boxes)
b2A1: CAO (bottom boxes)

c1B1: Follow through candidate's value provided that the M mark was earned in (b)
c2B1: CAO on critical activities (B, G, J)

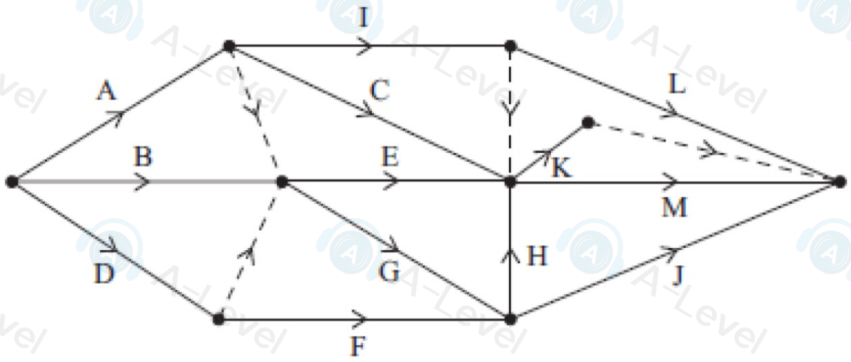
d1B1: Correct calculation with all three numbers present. An answer of 5 with no working scores B0

e1B1: **Correct calculation** seen then 3 – an answer of 3 with no working scores B0

f1M1: Not a cascade chart. 4 'workers' used at most and at least 9 unique activities placed
f1A1: 4 workers. All 10 activities present (just once). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
f2A1: 4 workers. All 10 activities present (just once). No errors

Activity	Duration	Time interval	IPA
A	6	0 -7	-
B	7	0 - 7	-
C	4	7 – 12	B
D	3	11 – 15	C
E	5	6 – 16	A
F	6	7 – 16	A, B
G	8	7 – 15	A, B
H	5	13 – 21	E, F
I	2	15 – 21	D, G
J	6	15 – 21	D, G

g1B1: Correctly stating the activities that are now critical (A, E and H) – no extras
g2B1: Correctly stating new project completion time (22 – no units required)

Question Number	Scheme	Marks
6.(a)	e.g.  	M1 A1 (A B D I C F) A1 (E G H J) A1 (K L M) A1 **(5)**
(b)	Activities B, G, H and M are critical	B1 **(1)**
(c)	At time 9.5, activities G, C, E and I must all be happening therefore a minimum of 4 workers is required	M1 A1 **(2)**
(d)	$2 \leqslant x \leqslant 6$	B2, 1, 0 **(2)**
		10 marks

Notes for Question 6

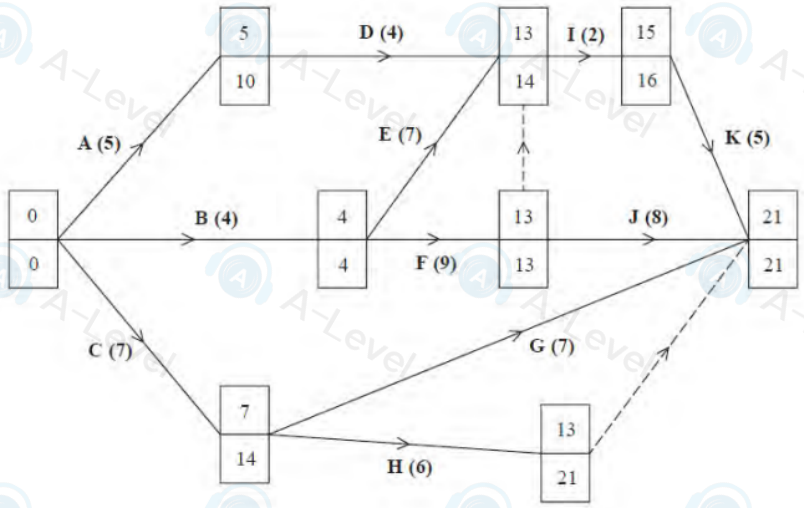
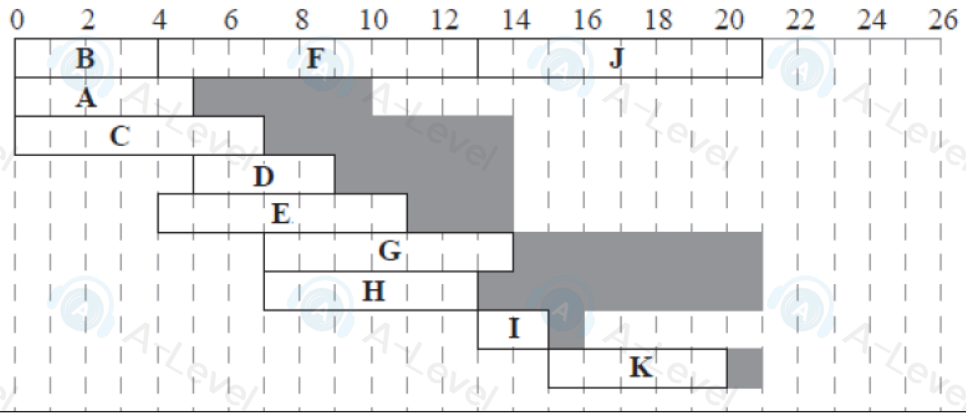
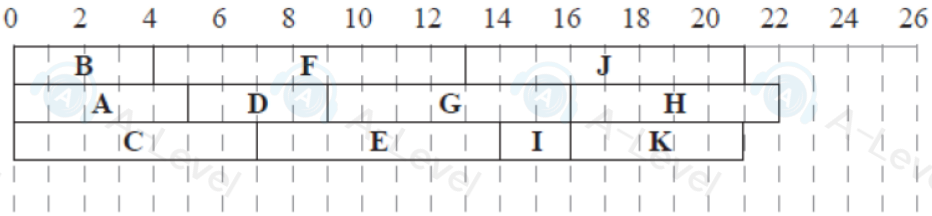
Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. 'J dealt with correctly' requires the correct precedences for this activity, i.e. F and G labelled correctly and leading into the same node and J starting from that node but do not consider the end event for J. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity F is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be). **Ignore missing arrows on the activities for the first four marks only**

a1M1	Nine activities (labelled on arc), one start and at least two dummies placed
a1A1	Activities A, B, D, 1st two dummies (including correct arrows), I, C and F dealt with correctly
a2A1	Activities E, G, H and J dealt with correctly
a3A1	Activities K, L and M dealt with correctly (so dummy at the end of I required + arrow)
a4A1	CSO – all arrows correctly placed for each activity with one finish and exactly four dummies (so uniqueness dummy for K/M required)

Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.
Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned. Note that this answer is not unique

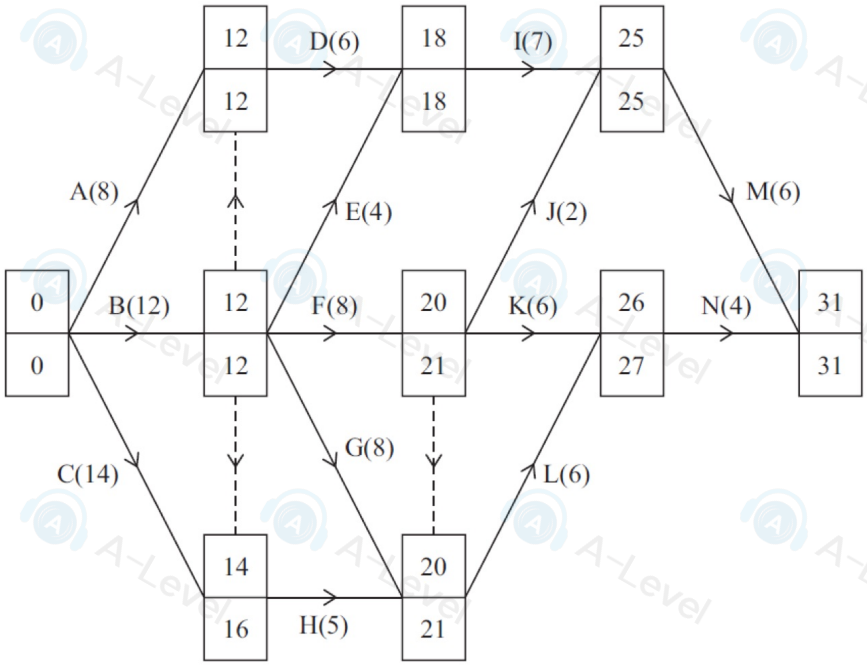
b1B1	CAO (B, G, H and M and no others)
c1M1	Either a statement with the correct number of workers (4) and stating the correct activities (G, C, E and I) with any numerical time stated **or** the correct number of workers (4) and a time in the interval $9 \leqslant t \leqslant 10$ – mark the numerical value only not their use of the words 'day/time' (or equivalent)
c1A1	A completely correct statement with details of both time **and** activities. Candidates must give a time within the correct interval of $9 < t < 10$, e.g. 9.5 (or 'on/during hour 10') and state the correct activities (G, C, E and I). Please note the strict inequalities for the time interval (e.g. implying a time of 10 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 9 – 10 or 'between 9 and 10' is A0). A completely correct statement with an additional incorrect statement scores A0 (so do not ignore subsequent working)
d1B1	For either $x \geqslant 2$ or $x \leqslant 6$ (must be x not L)
d2B1	CAO ($2 \leqslant x \leqslant 6$) – SC in (d) B1B0 for $2 < x < 6$

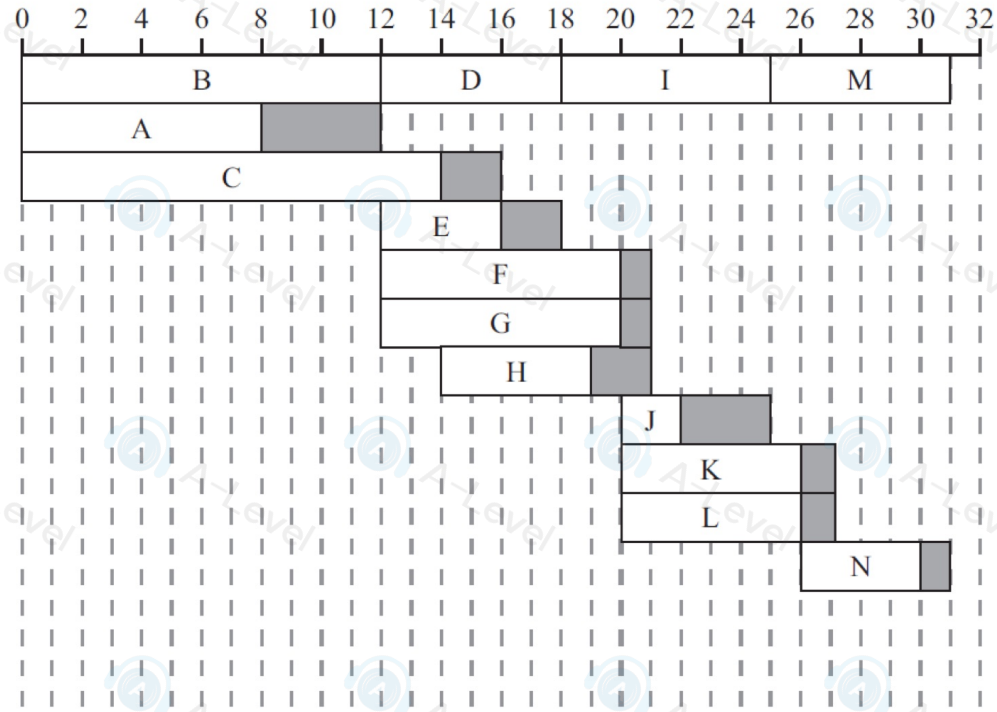
Question Number	Scheme	Marks
6.		
(a) (i)	The dummy from event 5 to event 6 is needed to show that J depends on F but I depends on D, E and F	B1
(ii)	The dummy from event 7 to event 9 is because activities G and H must be able to be described uniquely in terms of the events at each end	B1 **(2)**
(b)		M1 A1 M1 A1 **(4)**
(c)	21 (hours)	B1 **(1)**
(d)	$\frac{64}{21} \approx 3.048$ so at least 4 workers required	M1 A1 **(2)**
(e)		M1 A1 M1 A1 **(4)**
(f)	e.g. 	M1 A1 A1 **(3)**
		16 marks

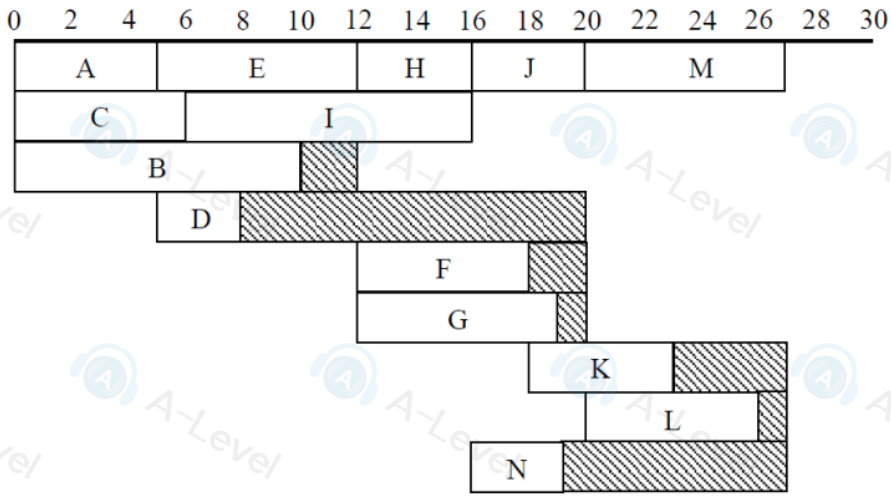
Question Number	Scheme	Marks
	Notes:	
a1B1	Cao – all relevant activities must be referred to - so activities I, J, F and either D or E must be mentioned.	
a2B1	Cao – mention of describing activities uniquely in terms of the events at each end.	
b1M1	All top boxes complete, values generally increasing from left to right, condone one 'rogue'	
b1A1	Cao on top boxes	
b2M1	All bottom boxes complete, values generally decreasing right to left, condone one 'rogue'	
b2A1	Cao on bottom boxes	
c1B1	Cao	
d1M1	Attempt to find lower bound: [55 – 73 / their finish time] or [sum of the activities / their finish time]	
d1A1	Cao – correct calculation seen then 4.	
e1M1	At least 8 activities added including 5 floats. Scheduling diagram scores M0.	
e1A1	Critical activites dealt with correctly and four other non-critical activities dealt with correctly.	
e2M1	All 11 activities including all 8 floats	
e2A1	Cao	
f1M1	Not a cascade chart. 3 workers used and at least 9 activities placed.	
f1A1	3 workers, All 11 activities present (just once). Condone one error either precedence or activity length.	
f2A1	3 workers. All 11 activities present (just once). No errors.	

Question	Scheme	Marks
7. (a)		B1 (DEFGH) B1 (IJK) B1 (LMN)
		(3)
(b)		M1 A1 M1 A1
		(4)

Activity	IPA	Activity	IPA
A	-	H	B C
B	-	I	D E
C	-	J	F
D	A B	K	F
E	B	L	F G H
F	B	M	I J
G	B	N	K L



(c)	Lower bound = (8+12+14+6+4+8+8+5+7+2+6+6+6+4)/31 = 96/31 = 3.1 Therefore 4 workers required	**M1** **A1**
		(2)
(d)		**M1** **A1** **A1** **A1**
		(4)
(e)	At $13 < t < 14$ activities D, C, F and G must be happening and at $16 < t < 17$ activities D, F, G and H must be happening. Also activity E must be happening at either $13 < t < 14$ or $16 < t < 17$ so a total of five workers	**M1** **A1** **A1**
		(3)
		(16 marks)

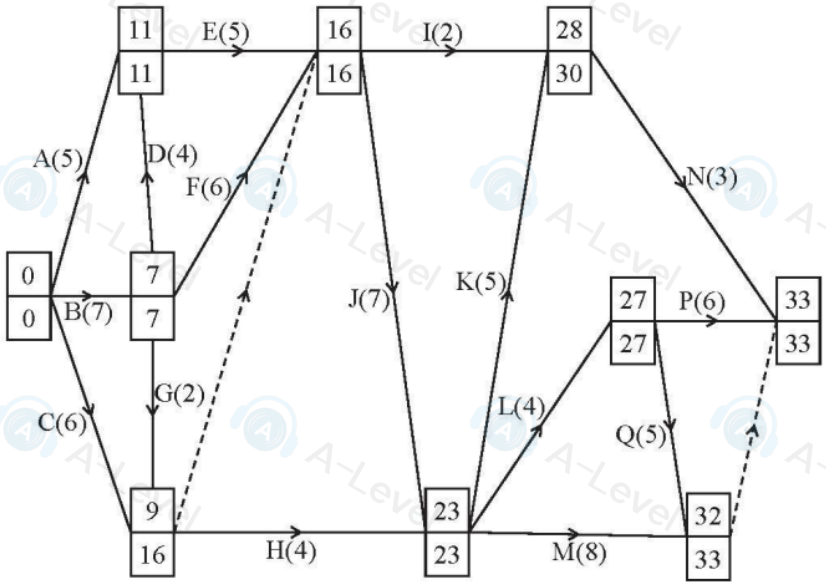
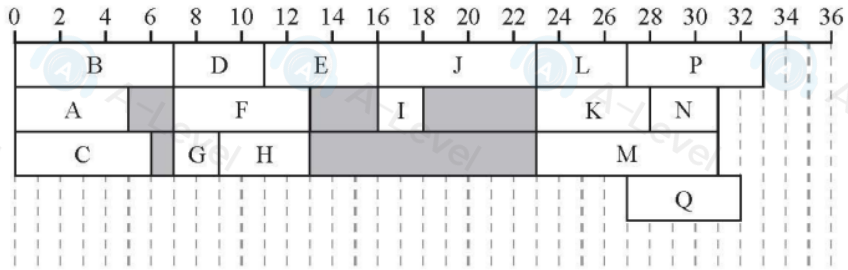
Question Number	Scheme	Marks
2.(a)	$w = 20, x = 6, y = 12, z = 10$	B4,3,2,1,0 **(4)**
(b)		M1 A1 M1 A1 **(4)**
(c)	Minimum workers is 4 activities H, I, F and G	M1
	together with 14 < time < 16	A1 **(2)**
		10 marks

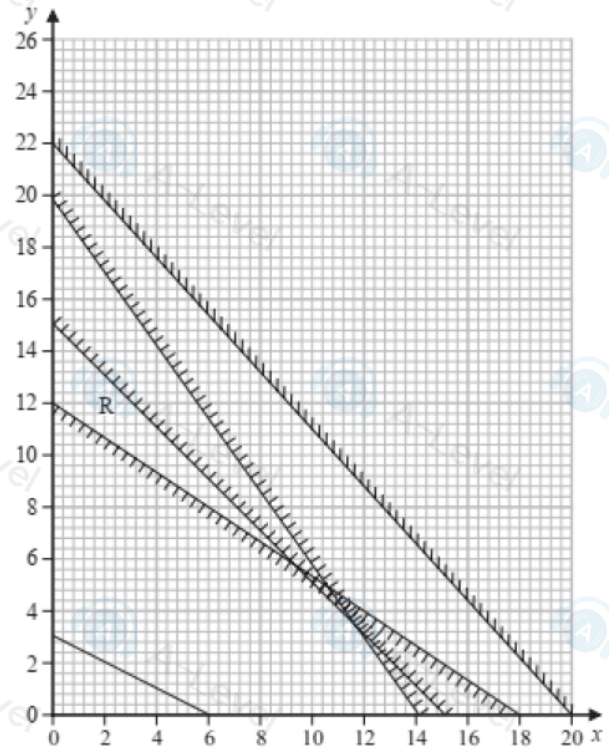
Notes for Question 2

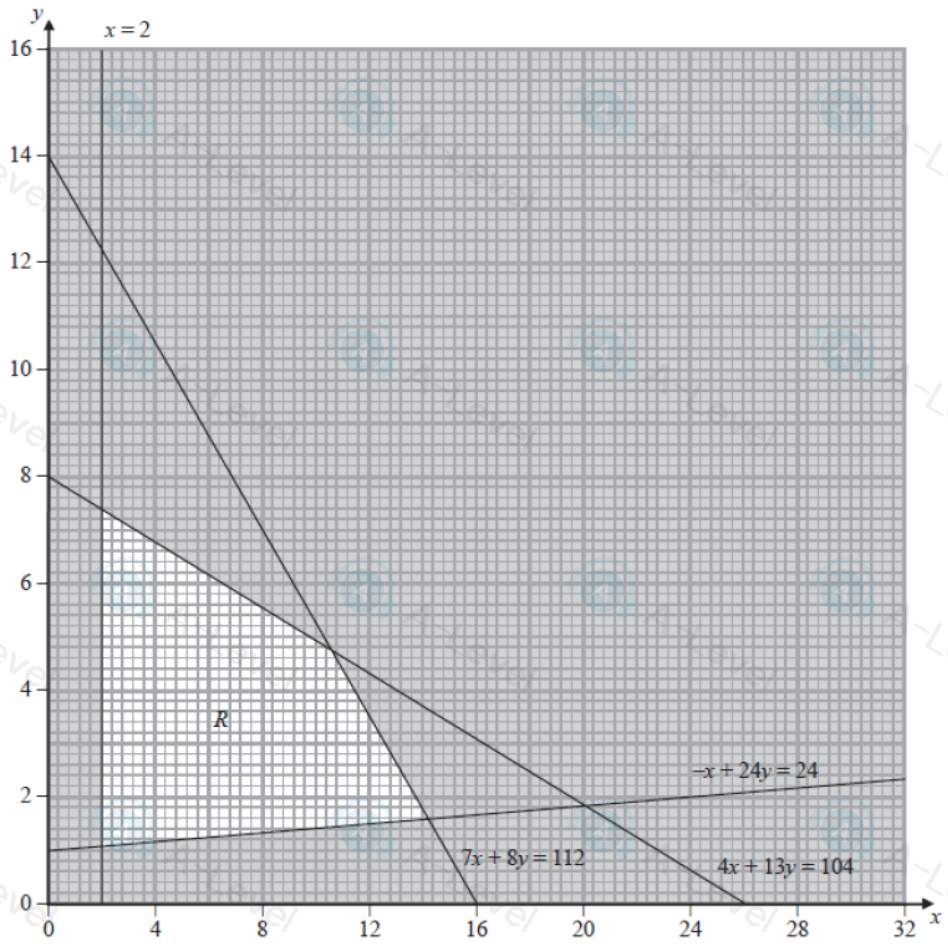
a1B1: Any one correct
a2B1: Any two correct
a3B1: Any three correct
a4B1: All four correct

b1M1: At least ten activities including at least four floats. A scheduling diagram scores M0
b1A1: The critical activities dealt with correctly and appearing just once (A, C, E, H, I, J and M) and 3 non-critical activities dealt with correctly
b2M1: Any five non-critical activities **correct**
b2A1: CSO – completely correct Gantt chart (exactly fourteen activities just once)

c1M1: Either a statement with the correct number of workers (4) and the correct activities (H, I, F and G) with any time stated **or** the correct number of workers (4) and a correct time
c1A1: A completely correct statement with details of both time and activities. Candidates only need to give a time within the correct interval of 14 < time < 16. Please note the strict inequalities for the time interval (e.g. implying a time of 14 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 15 – 16 is A0). Allow for example, 'on day 16' as equivalent to 15 < time < 16, and condone mention of 'day' if a correct time is given but 'at day 16' is A0

Question Number	Scheme	Marks
1. (a)		M1 A1 M1 A1 **(4)**
(b)	Activity H can be delayed by 23 – 9 – 4 = 10 (days)	B1ft **(1)**
(c)	Lower bound = $\frac{5+7+6+\ldots+6+5}{33} = \frac{79}{33} = 2.393\ldots = 3$ workers	M1 A1 **(2)**
(d)	e.g. 	M1 A1 A1 **(3)**
		10 marks

Question number	Scheme	Marks
6. (a)	Minimise ($P =$) $300x + 400y + 400z$	B1
	Subject to:	
	$275x + 200y + 100z \leqslant 5500$ ($\Rightarrow 11x + 8y + 4z \leqslant 220$)	B1
	$5x + 2y + 3z \leqslant 70$	B1
	$\frac{x}{15} + \frac{y}{20} + \frac{z}{30} \leqslant 1 \Rightarrow 4x + 3y + 2z \leqslant 60$	M1 A1
	$x + y + z \geqslant 18$ $(x, y, z \geqslant 0)$	B1 **(6)**
(b)	e.g Martin makes apple cakes and chocolate cakes in the ratio of 2:1 e.g. for every one chocolate cake that Martin makes he has to make 2 apple cakes (assuming that he makes chocolate/apple cakes) e.g. Martin should make twice as many apple cakes as chocolate cakes e.g. The number of apple cakes that Martin makes should be double the number of chocolate cakes he makes	B1 **(1)**
(c)		B1 B1 B1 B1 **(4)**
(d)	Drawing an objective line accept reciprocal gradient	M1
	Correct objective line	A1
	Martin should make 9 carrot cakes, 6 apple cakes and 3 chocolate cakes	A1
	Minimum amount of sugar is 6300 grams (or 6.3 kg)	A1 **(4)**
(e)	Martin has 1525 grams of flour remaining	B1
	Martin also has 4 eggs remaining	B1 **(2)**
		(17 marks)

Question Number	Scheme	Marks
8. (a)	$20x+65y\leq 520$ or $4x+13y\leq 104$ (oe)	B1 **(1)**
(b)		B1 B1 B1 DB1 (R) **(4)**
(c)	e.g. $(P=)\,2x+3y$	B1 **(1)**

Question Number	Scheme	Marks
(d)	Drawing an objective line accept reciprocal gradient Correct objective line minimum length equivalent to (0, 1) to (1.5, 0) V correctly labelled	M1 A1 A1 **(3)**
(e)	V$\left(\frac{624}{59}, \frac{280}{59}\right)$	M1 A1 **(2)**
(f)	Testing integer solutions around V, $x = 11$ and $y = 4$ is optimal integer solution, so they should buy 11 standard containers and 4 deluxe containers Cost is (£) 480	M1 A1 B1 **(3)** **14 marks**

Notes for Question 8

a1B1: CAO – accept any exact equivalent inequality (isw if simplified incorrectly)

In (b):
$7x+8y=112$ must pass within one small square of its intersection with the axes – (0, 14) and (16, 0)
$20x+65y=520$ must pass within one small square of its intersection with the axes – (0, 8) and (26, 0)
$-x+24y=24$ must be sufficiently long to define the feasible region and pass within one small square of (0, 1) and (24, 2) if extended
$x=2$ must pass within one small square of (2, 0) and (2, 8)

b1B1: Any two lines correctly drawn
b2B1: Any three lines correctly drawn
b3B1: All four lines correctly drawn
b4DB1: Region, R, correctly labelled – not just implied by shading – dependent on scoring the first three marks in this part

c1B1: CAO - $k(2x+3y)$ where $k \in \mathbb{R}$ – condone equal to P or equal to a constant

d1M1: Drawing either the correct objective line **or** their objective line (based on their answer to (c)) **or** the reciprocal of the correct objective line **or** the reciprocal of their objective line – if their line is shorter than the length equivalent to that of the line from (0, 1) to (1.5, 0) then M0. Line must be correct to within one small square if extended from axis to axis
d1A1: Drawing the correct objective line – same condition that the line must be correct to within one small square if extended from axis to axis
d2A1: Correct V labelled clearly on their graph – **please note that this mark is dependent on scoring at least B1B1B1B0 in (b) and the two previous marks in this part** – by clearly labelled the vertex should either be labelled 'V' or circled or clearly distinguishable from the other three (but A0 if not clear e.g. other vertices circled too)

e1M1: Simultaneous equations being used to find V. **Must have scored at least B1B1B0B0 in (b) and candidates must have labelled one of their vertices as V (oe – see above)**. Must be solving either the pair

Question Number	Scheme	Marks
	of simultaneous equations $20x+65y=520$ and $7x+8y=112$ **or** $7x+8y=112$ and $-x+24y=24$. Must be a correct method to solve simultaneous equations and must arrive at $x = \ldots$ and $y = \ldots$ but allow slips/errors. **This mark can also be awarded for the correct exact coordinates stated with no working provided B1B1B0B0 in (b) and a vertex labelled as V** e1A1: Correct exact coordinates of the correct V derived with working (not just stated) as either $\left(\frac{624}{59},\frac{280}{59}\right)$ or $\left(10\frac{34}{59},4\frac{44}{59}\right)$ or stated just in terms of x and y. **Note that this mark is dependent on B1B1B1B0 scored in (b) and all three marks in (d)**. ISW if correct exact values seen followed by decimal approximations f1M1: Testing any two of (11, 4) or (9, 5) or (10, 5) or (10, 4) or (11, 5) in a correct objective function **or** the correct pair of inequalities. Note candidates may reject a point after testing in only one correct inequality which is acceptable – **this mark is not dependent on any previous mark** f1A1: CSO (**all previous 12 marks must have been awarded**) – must have tested (11, 4) in the correct objective function **or** correct pair of inequalities – accept $x = 11$ and $y = 4$ or stated as a pair of coordinates f1B1: CAO – this mark is not dependent on any previous mark and condone lack of units	

Question Number	Scheme	Marks
4.(a)	$y \leqslant 2x$ and $3x+y \leqslant 30$	B1
	Correct method for finding the third boundary of the feasible region	M1
	$5y \geqslant 6x+10$	A1 **(3)**
(b)	(6, 12)	B1
	Solving simultaneous equations to find the other two vertices	M1
	$\left(\frac{5}{2},5\right)$ and $\left(\frac{20}{3},10\right)$	A1 **(3)**
(c)	$\left(\frac{5}{2},5\right) \rightarrow P=20$ $\left(\frac{20}{3},10\right) \rightarrow P=\frac{130}{3}$ $(6,12) \rightarrow P=48$	M1 A1
	Optimal vertex is (6,12) with $P=48$	A1 **(3)**
(d)	$12+12k \square 2\left(\frac{20}{3}\right)+10k$ where $\square$ is any inequality or equals	M1
	$k \geqslant \frac{2}{3}$	A1 **(2)**
		11 marks

Notes for Question 4

a1B1: Both correct inequalities: $y \leqslant 2x$ and $3x+y \leqslant 30$ (or equivalent if they have been rearranged)

a1M1: Correct equation for the third boundary of the feasible region, e.g. $\frac{y-20}{14-20}=\frac{x-15}{10-15}$ would score **M1** (so does not need to be simplified). Condone any inequality sign used (provided that if the inequality sign was replaced with an 'equals' then the equation ($5y = 6x + 10$) would be correct). ISW once the correct equation has been seen (so condone the correct unsimplified equation if not simplified correctly)

a1A1: CAO for third inequality – must be three terms only but need not be further simplified (e.g. $12x-10y+20 \leqslant 0$, $y \geqslant 1.2x+2$ etc. scores **A1**)

b1B1: (6, 12) **or** $x = 6$, $y = 12$

b1M1: Correct method for solving their simultaneous equations (so $y = 2x$ with their $5y = 6x + 10$ **or** $3x + y = 30$ with their $5y = 6x + 10$) to find both the x and y-coordinates of at least one of the other two vertices (can be implied by either vertex stated correctly – condone non-exact values (to at least 3 sf) for this mark if no working seen)

b1A1: CAO (2.5, 5) **and** $\left(\frac{20}{3},10\right)$ (must be exact, so accept $(6\frac{2}{3},10)$, $(6.\dot{6},10)$, etc. but not (6.6666…, 10))

c1M1: Testing their three **vertices** (not just points in the FR) in the correct objective function ($P = 2x + 3y$). Condone one slip only when applying the objective function to one of their three vertices (so they must apply the correct objective to at least two of their three points but condone a single slip in the third)

c1A1: At least two correct (therefore exact) points tested correctly (so at least two correct values of P explicitly stated – allow awrt 43.3 when testing (20/3, 10))

c2A1: CSO all three correct (therefore exact) points tested correctly (so all three correct values of P explicitly seen – allow awrt 43.3 when testing (20/3, 10)) with a clear indication of which is the optimal vertex (this can be achieved by either making it clear that (6, 12) is the optimal point **or** that $P = 48$ is the maximum (as one implies the other))

Question Number	Scheme	Marks
5.(a)	$\frac{1}{2}(x+y+z) \leqslant y \quad (\Rightarrow x-y+z \leqslant 0)$	M1
	$\frac{3}{20}(x+y+z) \geqslant z \quad (\Rightarrow 3x+3y-17z \geqslant 0)$	M1
	$2x = 5z$	B1
	$5x+12y+15z \leqslant 834$	B1
	Eliminating z from the objective $x+y+z$ and at least one correct constraint or states the objective and eliminates z from at least two correct constraints	M1
	Maximise ($P=$) $1.4x+y$ subject to $11x+12y \leqslant 834$ $7x-5y \leqslant 0$ $19x-15y \leqslant 0$ $(x \geqslant 0,\ y \geqslant 0)$	A1 A1 **(7)**
(b)(i)	Substitute $y=42$ into LP gives $x \leqslant 30,\ x \leqslant 30,\ x \leqslant \frac{630}{19}$ which implies that $x=30$	M1
	Total number of reams ordered is $1.4(30)+42=84$	A1
(b)(ii)	12 reams of graph paper ordered	A1 **(3)**
		10 marks
	Notes for Question 5	
a1M1	$\frac{1}{2}(x+y+z) \bullet y$ where $\bullet$ is any inequality or equals sign (accept any equivalent form)	
a2M1	$\frac{3}{20}(x+y+z) \bullet z$ where $\bullet$ is any inequality or equals sign (accept any equivalent form)	
a1B1	Correct relationship between x and z (accept any equivalent form)	
a2B1	CAO ($5x+12y+15z \leqslant 834$)	
a3M1	Either eliminating z (using either $2x=5z$ or $2z=5x$ oe) from the correct objective and at least one correct constraint or states objective in the form ($P=$) $x+y+z$ and eliminates z from at least two correct constraints	
a1A1	Either two correct inequalities in x and y only **or** one correct inequality and the correct objective (but do not need 'maximise' or 'max') in x and y only – note the correct inequality must have integer coefficients (accept equivalent forms for the objective e.g. $5P=7x+5y$ (**must include 5*P* =**) if $P=7x+5y$ stated do not ISW)	
a2A1	Correct LP formulation in x and y including 'maximise' or 'max' with objective (condone if previously stated) – note that all constraints must have integer coefficients (condone lack of $x \geqslant 0,\ y \geqslant 0$). Do not ISW if any incorrect simplifications seen.	
bi1M1	Substituting $y=42$ into at least two of their constraints (must only be in terms of x and y) leading to a value of x (if they over simplify and assume that 42 is exactly ½ of the total number of reams, they must still substitute into at least 2 constraints)	
bi1A1	CAO (84) – must come from correct working	
bii1A1	CAO (12) – must come from correct working	

Question Number	Scheme	Marks
7. (a)	Maximise $0.75x+1.2y+1.45z$	B1
	Subject to $x+z<200$	B1
	$5y \geq 2x$	M1 A1
	$\frac{3}{4}(x+y+z) \geq y \Rightarrow 3x+3z \geq y$ $(x,y,z \geq 0)$	M1 A1 **(6)**
(b)	$x=100, z=25$ leading to $40 \leq y \leq 375$ Minimum profit (£)159.25, Maximum profit (£)561.25	M1 A1 A1 **(3)**
		9 marks

Notes for Question 7

a1B1: Expression correct (or $75x+120y+145z$) **together with** 'maximise' or 'max' but not 'maximum' – isw if coefficients are subsequently simpified but either $0.75x+1.2y+1.45z$ or $75x+120y+145z$ **together with** 'maximise' or 'max' must be seen at some point for this mark to be awarded

a2B1: CAO $(x+z<200)$

a1M1: Correct method: $5y \square 2x$ (oe e.g. $2.5y \square x$) where $\square$ is any inequality or equals. An exact equivalent answer (with or without integer coefficients) can score M1 **or** for $2y \geq 5x$ (oe)

a1A1: CAO $(5y \geq 2x$ oe) - answer must have integer coefficients with like terms collected i.e. $k(5y \geq 2x)$ for any positive integer k - the correct answer with no working can score M1 A1

a2M1: $\frac{3}{4}(x+y+z) \square y$ (oe) where $\square$ is any inequality or equals. The bracket must be present or implied by later working. An exact equivalent answer (with or without integer coefficients) with no working can score M1. Accept equivalent fractions or decimals for 3/4 but not 75% (unless later converted to a correct fraction/decimal)

a2A1: CAO $(3x+3z \geq y)$ (oe) - answer must have integer coefficients with like terms collected i.e. $k(3x+3z \geq y)$ for any positive integer k - the correct answer with no working can score M1 A1

b1M1: Substituting $x=100$ **and** $z=25$ into their inequalities and obtaining **either** two values of y **or** two inequalities for y **or** for either correct answer of 159.25 **or** 561.25 seen

b1A1: CAO - $40 \leq y \leq 375$ (oe) **or** $y=40$ **and** $y=375$ (must come from correct working/inequalities)

b2A1: CAO on minimum and maximum profit (must come from correct working) – condone lack of or incorrect units. Furthermore, candidates do not need to explicitly state which is the minimum and which is the maximum profit for this mark