Question Number	Scheme	Notes	Marks
9.	$f(n) = 7^n(3n+1) - 1$ is a multiple of 9	$u_1 = 2, u_2 = 6, u_{n+2} = 3u_{n+1} - 2u_n \Rightarrow u_n = 2(2^n - 1)$	
(i) **Way 1**	$f(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$f(1) = 27$ is the minimum	B1
	$f(k+1) - f(k) = 7^{k+1}(3(k+1)+1) - 1 - (7^k(3k+1) - 1)$	Attempts $f(k+1) - f(k)$	M1
		A correct expression for $f(k+1)$	A1
	$= 7^{k+1}(3k+4) - 1 - 7^k(3k+1) + 1 = 7^k(21k+28) - 7^k(3k+1)$		
	$= 18k(7^k) + 27(7^k)$ or $7^k(18k+27)$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$f(k+1) = 9(7^k)(2k+3) + 7^k(3k+1) - 1$ or $f(k+1) = 18k(7^k) + 27(7^k) + f(k)$	Correct algebra leading to either e.g. $f(k+1) = 9(7^k)(2k+3) + 7^k(3k+1) - 1$ or $f(k+1) = 18k(7^k) + 27(7^k) + f(k)$	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
(i) **Way 2**	$f(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$f(1) = 27$ is the minimum	B1
	$f(k+1) = 7^{k+1}(3(k+1)+1) - 1$	Attempts $f(k+1)$	M1
		A correct expression for $f(k+1)$	A1
	$= 7^{k+1}(3k+4) - 1 = 7^k(21k+28) - 1$		
	$= 18k(7^k) + 27(7^k) + 7^k(3k+1) - 1$ **or** $= (7^k)(18k+27) + 7^k(3k+1) - 1$ **or** $= 9(7^k)(2k+3) + 7^k(3k+1) - 1$	**dependent on the previous M mark** Uses correct algebra to express $f(k+1) = g(k) + 7^k(3k+1) - 1$ or $f(k+1) = g(k) + f(k)$ where each term in $g(k)$ is an obvious multiple of 9	dM1
		Correct algebra leading to either e.g. $f(k+1) = 9(7^k)(2k+3) + 7^k(3k+1) - 1$ or $f(k+1) = 18k(7^k) + 27(7^k) + f(k)$	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
(ii)	$\{n=1,\}$ $u_1 = 2(2^1 - 1) = 2$;	Checks that the general formula works for either u_1 or u_2	M1
	$\{n=2,\}$ $u_2 = 2(2^2 - 1) = 6$	Checks that the general formula works for both u_1 and u_2	A1
	$\{u_{k+2} = 3u_{k+1} - 2u_k \Rightarrow\}$ $u_{k+2} = 3(2(2^{k+1} - 1)) - 2(2(2^k - 1))$	Finds u_{k+2} by attempting to substitute $u_{k+1} = 2(2^{k+1} - 1)$ and $u_k = 2(2^k - 1)$ into $u_{k+2} = 3u_{k+1} - 2u_k$ Condone one slip	M1
	$\{u_{k+2}\} = 6(2^{k+1}) - 6 - 4(2^k) + 4$		
	$\{u_{k+2}\} = 3(2^{k+2}) - 2^{k+2} - 2$	Valid evidence of working in the same power of 2	M1
	$= 2(2^{k+2}) - 2 = 2(2^{k+2} - 1)$	***Uses algebra*** in a complete method to achieve this result with no errors	A1
	If the result is true for $n = k$ and for $n = k+1$, then it is true for $n = k+2$. As the result has been shown to be true for $n = 1$ and $n = 2$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
			12

		Question 9 Notes
9. (i)	**Note**	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (i)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	Shows $\text{f}(k+1)-\text{f}(k)=7^k(18k+27)$ **or** $\text{f}(k+1)-\text{f}(k)=9(7^k)(2k+3)$ and writing if $\text{f}(k+1)-\text{f}(k)=9(7^k)(2k+3)$ o.e. is a multiple of 9 then $\text{f}(k+1)$ is a multiple of 9 is acceptable for the penultimate A mark in part (i). This means that the final A mark can potentially be available.
	Note	Only showing $\text{f}(k+1)=\ 7\text{f}(k)\ +6+21(7^k)$ (see Way 4) does not get the final dM mark because $6+21(7^k)$ is not an obvious multiple of 9
	Note	Allow dM1 for obtaining e.g. $\text{f}(k+1)-\text{f}(k)=18k(7^k)-27(7^k)$ or $\text{f}(k+1)-\text{f}(k)=7^k(18k-27)$
	Note	Allow dM1 for obtaining $\text{f}(k+1)=18k(7^k)-27(7^k)+7^k(3k+1)-1$ or $\text{f}(k+1)=9(7^k)(2k-3)+\text{f}(k)$
(ii)	**Note**	**1st M1:** At least one check is correct. **1st A1:** Both checks are correct • Check 1: Shows $u_1=2$ by writing an intermediate step of e.g. $2(2^1-1)$ or 2×1 • Check 2: Shows $u_2=6$ by writing an intermediate step of e.g. $2(2^2-1)$ or 2×3
	Note	Ignore $u_3=3u_2-2u_1=3(6)-2(2)=14$ as part of their solution to (ii)
	Note	Ignore $\{n=3,\}\ u_2=2(2^3-1)=14$ as part of their solution to (ii)
	Note	Valid evidence of working in the same power of 2 includes: • $6(2^{k+1})-4(2^k)\rightarrow6(2^{k+1})-2(2^{k+1})$ or $2(3(2^{k+1})-2^{k+1})$ • $3(2(2^{k+1}))-2(2(2^k))\ \rightarrow3(2^{k+2})-(2^{k+2})$ • $3(2(2^{k+1}))-2(2(2^k))\ \rightarrow12(2^k)-4(2^k)$ • $6(2^{k+1})-4(2^k)\rightarrow8(2^k)$ (by implication) • $6(2^{k+1})-4(2^k)\rightarrow4(2^{k+1})$ (by implication)
	Note	Writing $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))=2(2^{k+2}-1)$ is 2nd M1, 3rd M0, 2nd A0
	Note	Showing {RHS = } $u_{k+2}=2(2^{k+2}-1)=8(2^k)-2$ and writing {LHS = } $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$ and using valid algebra to show that $u_{k+2}=8(2^k)-2$ {= RHS} is fine for the 2nd M, 3rd M and 2nd A marks
	Note	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (ii)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	"Assume for $n=k$, $u_k=2(2^k-1)$ and for $n=k+1$, $u_{k+1}=2(2^{k+1}-1)$" satisfies the requirement "true for $n=k$ and $n=k+1$"
	Note	"For $n\in\mathbb{Z}^+$, $u_n=2(2^n-1)$ " satisfies the requirement "true for all *n*"
	Note	Full marks in (ii) can be obtained for an equivalent proof where e.g. • $n=k,\ n=k+1,\ \rightarrow n=k-2,\ n=k-1$; i.e. $k\equiv k-2$ • $n=k,\ n=k+1,\ \rightarrow n=k-1,\ n=k$; i.e. $k\equiv k-1$
(i), (ii)	**Note**	Allow as part of their conclusion "true for all positive values of *n*"
	Note	Allow as part of their conclusion "true for all values of *n*"
	Note	Allow as part of their conclusion "true for all $n\in\mathbb{N}$"
	Note	Condone referring to *n* as any integer in their conclusion for the final A1
	Note	Condone $n\in\mathbb{Z}^*$ as part of their conclusion for the final A1
	Note	Referring to *n* as a real number their conclusion is final A0

Question Number	Scheme	Notes	Marks
9.	$f(n) = 7^n(3n+1) - 1$ is a multiple of 9; $P \in \mathbb{Z}^+$		
(i)	$f(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$f(1) = 27$ is the minimum	B1
Way 3	$f(k+1) - (9P+1)f(k)$	Attempts $f(k+1) - (9P+1)f(k)$	M1
	$= \underline{7^{k+1}(3(k+1)+1) - 1} - (9P+1)(7^k(3k+1)-1)$	A correct expression for $\underline{f(k+1)}$	A1
	$= 7^k(21k+28-(9P+1)(3k+1)) - 1 + 9P + 1$		
	$= 7^k(21k+28-(27Pk+9P+3k+1)) - 1 + 9P + 1$		
	$= 7^k(21k+28-27Pk-9P-3k-1) + 9P$		
	$= 7^k(18k-27Pk-9P+27) + 9P$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$f(k+1) = 7^k(18k-27PK-9P+27) + 9P + (9P+1)f(k)$	Achieves a correct result for $f(k+1) = \ldots$	A1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	Note: $P = 0 \Rightarrow f(k+1) - f(k) = 7^k(18k+27)$ $P = 1 \Rightarrow f(k+1) - 10f(k) = 7^k(18-9k) + 9$ $P = 2 \Rightarrow f(k+1) - 19f(k) = 7^k(9-36k) + 18$ $P = 3 \Rightarrow f(k+1) - 28f(k) = 7^k(-63k) + 27 = 27 - 9k(7^{k+1})$		

Question Number	Scheme	Notes	Marks
9.	$f(n) = 7^n(3n+1) - 1$ is a multiple of 9		
(i) **Way 4**	$f(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$f(1) = 27$ is the minimum	B1
	$f(k+1) = 7^{k+1}(3(k+1)+1) - 1$	Attempts $f(k+1)$	M1
		A correct expression for $f(k+1)$	A1
	$= 7(7^k)(3k+3+1) - 1$		
	$= 7(7^k)(3k+1) + 3(7)(7^k) - 1$		
	$= 7[(7^k)(3k+1) - 1] + 7 + 21(7^k) - 1$ $= 7f(k) + 6 + 21(7^k)$ Let $g(n) = 6 + 21(7^n)$ $g(1) = 6 + 21(7^1) = 153$ {is a multiple of 9} {Assume the result is true for $n = k$} $g(k+1) = 6 + 21(7^{k+1})$	**dependent on the previous M mark** Uses correct algebra to express $f(k+1) = \alpha(7^k(3k+1) - 1) + g(k)$ or $f(k+1) = \alpha f(k) + g(k);\ \alpha \neq 0$ and uses correct algebra to achieve an expression for $g(k+1)$ where each term is an obvious multiple of 9	M1
	$= 6 + 147(7^k)$ $= 6 + 21(7^k) + 126(7^k)$ or $= g(k) + 9(14)(7^k)$	Correct algebra leading to $f(k+1) = 7f(k) + 6 + 21(7^k)$ o.e. and $g(k+1) = 6 + 21(7^k) + 126(7^k)$ where $g(n) = 6 + 21(7^n)$	A1
	Proves that $g(n) = 6 + 21(7^n)$ is a multiple of 9 and proves that for $f(n)$ if the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$</u>. As the result has been shown to be <u>true for $n = 1$</u>, then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	Note: An alternative Way 4 method shows • $f(k+1) = 7f(k) + 6 + 21(7^k) = 7f(k) + 9(7^k + 1) + 3(7^k) - 3$ • Defines $g(n) = 3(7^n) - 3$ and proceeds to show that $g(n)$ is also a multiple of 9		

Question Number	Scheme	Notes	Marks
3.	Required to prove by induction the result $\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} \quad \frac{1}{(n+1)(n+2)}, \; n$		
Way 1	$n=1:\text{LHS} = \frac{1}{3}$, $\text{RHS} = \frac{1}{2} \quad \frac{1}{(2)(3)} = \frac{1}{3}$	Shows or states $\text{LHS} = \frac{1}{3}$ **and shows either** $\text{RHS} = \frac{1}{2} \quad \frac{1}{(1+1)(2+1)} = \frac{1}{3}$ **or** $\text{RHS} = \frac{1}{2} \quad \frac{1}{(2)(3)} = \frac{1}{3}$ **or** $\text{RHS} = \frac{1}{2} \quad \frac{1}{6} = \frac{1}{3}$	B1
	(Assume the result is true for $n=k$)		
	$\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} \quad \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+1+1)(k+1+2)}$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= \frac{1}{2} \quad \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+2)(k+3)}$		
	$= \frac{1}{2} \quad \frac{(k+3)}{(k+1)(k+2)(k+3)} + \frac{2}{(k+1)(k+2)(k+3)}$ **or** $= \frac{1}{2} \quad \left(\frac{(k+3) \quad 2}{(k+1)(k+2)(k+3)}\right)$	**dependent on the previous M mark** Makes $(k+1)(k+2)(k+3)$ a common denominator for their second and third fractions	dM1
	$= \frac{1}{2} \quad \frac{1}{(k+2)(k+3)}$	Obtains $\frac{1}{2} \quad \frac{1}{(k+2)(k+3)}$ or $\frac{1}{2} \quad \frac{1}{(k+1+1)(k+1+2)}$ **by correct solution only**	A1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result <u>is true for all n</u> ( )		A1 **cso**
	Final A1 is dependent on all previous marks being scored in that part. It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.		**(5)**
			5
Way 2	**The M1dM1A1 marks for Alternative Way 2**		
	$\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} \quad \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+1+1)(k+1+2)}$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= \frac{(k+1)(k+2)(k+3) \quad 2(k+3) + 2(2)}{2(k+1)(k+2)(k+3)}$	**dependent on the previous M mark** Makes $2(k+1)(k+2)(k+3)$ a common denominator for their three fractions	dM1
	$= \frac{k^3+6k^2+9k+4}{2(k+1)(k+2)(k+3)} = \frac{(k+1)(k^2+5k+4)}{2(k+1)(k+2)(k+3)} = \frac{k^2+5k+4}{2(k+2)(k+3)} = \frac{(k+2)(k+3) \quad 2}{2(k+2)(k+3)}$		
	$= \frac{1}{2} \quad \frac{1}{(k+2)(k+3)}$	Obtains $\frac{1}{2} \quad \frac{1}{(k+2)(k+3)}$ or $\frac{1}{2} \quad \frac{1}{(k+1+1)(k+1+2)}$ **by correct solution only**	A1

		Question 3 Notes		
3.	**Note**	LHS = RHS by itself or LHS = RHS = $\frac{1}{3}$ is not sufficient for the 1st B1 mark.		
	Note Way 2	The 1st A1 can be obtained by e.g. using algebra to show that $\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)}$ gives $\frac{(k^2+5k+4)}{2(k+2)(k+3)}$ and by using algebra to show that $\frac{1}{2} \quad \frac{1}{(k+2)(k+3)}$ also gives $\frac{(k^2+5k+4)}{2(k+2)(k+3)}$		
	Note	Moving from $\frac{1}{2} \quad \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+2)(k+3)}$ to $\frac{1}{2} \quad \frac{1}{(k+2)(k+3)}$ ***with no intermediate working*** is 2nd M0 1st A0 2nd A0.		
Way 3	**The M1dM1A1 marks for Alternative Way 3**			
	$\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} \quad \frac{1}{(k+1)(k+2)} + \frac{2}{(k+1)(k+1+1)(k+1+2)}$		Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= \frac{1}{2} \quad \frac{1}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} \quad \frac{1}{(k+2)(k+3)}$		**dependent on the previous M mark** This step must be seen in Way 3	dM1
	$= \frac{1}{2} \quad \frac{1}{(k+2)(k+3)}$		Obtains $\frac{1}{2} \quad \frac{1}{(k+2)(k+3)}$ or $\frac{1}{2} \quad \frac{1}{(k+1+1)(k+1+2)}$ **by correct solution only**	A1

Question Number	Scheme	Notes	Marks
9.	(i) $\sum_{r=1}^{n}(4r^3 \quad 3r^2+r) = n^3(n+1)$; (ii) $f(n) = 5^{2n}+3n \quad 1$ is divisible by 9		
(i)	$n=1$: LHS $= 4 \quad 3+1 = \quad 2$, RHS $= 1^3(1+1) = 2$	Shows or states **both** LHS $= 2$ **and** RHS $= 2$ **or** states LHS $=$ RHS $= \quad 2$	B1
	(Assume the result is true for $n=k$)		
	$\sum_{r=1}^{k+1}(4r^3 \quad 3r^2+r) = k^3(k+1) + 4(k+1)^3 \quad 3(k+1)^2+(k+1)$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= (k+1)\left[k^3+4(k+1)^2 \quad 3(k+1)+1\right]$ or $(k+1)\left[k^3+4k^2+5k+2\right]$ or $(k+2)\left[k^3+3k^2+3k+1\right]$	**dependent on the previous M mark**. Takes out a factor of either $(k+1)$ or $(k+2)$	dM1
	$= (k+1)(k+1)(k+1)(k+2)$	**dependent on both the previous M marks**. Factorises out and obtains either $(k+1)(k+1)(....)$ or $(k+1)(k+2)(....)$	ddM1
	$= (k+1)^3(k+1+1)$ or $= (k+1)^3(k+2)$	Achieves this result with no errors.	A1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n ( )		A1 **cso**
	Note: Expanded quartic is $k^4+5k^3+9k^2+7k+2$		**6**
(ii) **Way 1**	$f(1) = 5^2+3 \quad 1 = 27$	$f(1) = 27$ is the minimum	B1
	$f(k+1) \quad f(k) = (5^{2(k+1)}+3(k+1) \quad 1) \quad (5^{2k}+3k \quad 1)$	Attempts $f(k+1) \quad f(k)$	M1
	$f(k+1) \quad f(k) = 24(5^{2k})+3$		
	$= 24(5^{2k}+3k \quad 1) \quad 9(8k \quad 3)$	$24(5^{2k}+3k \quad 1)$ or $24f(k)$	A1
	or $= 24(5^{2k}+3k \quad 1) \quad 72k+27$	$9(8k \quad 3)$ or $72k+27$	A1
	$f(k+1) = 24f(k) \quad 9(8k \quad 3)+f(k)$ or $f(k+1) = 24f(k) \quad 72k+27+f(k)$ or $f(k+1) = 25(5^{2k}+3k \quad 1) \quad 72k+27$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(5^{2k}+3k \quad 1)$	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$, As the result has been shown to be true for $n=1$, then the result is true for all n ( )		A1 **cso**
			(6)
(ii) **Way 2**	$f(1) = 5^2+3 \quad 1 = 27$	$f(1) = 27$ is the minimum	B1
	$f(k+1) = 5^{2(k+1)}+3(k+1) \quad 1$	Attempts $f(k+1)$	M1
	$f(k+1) = 25(5^{2k})+3k+2$		
	$= 25(5^{2k}+3k \quad 1) \quad 9(8k \quad 3)$	$25(5^{2k}+3k \quad 1)$ or $25f(k)$	A1
	or $= 25(5^{2k}+3k \quad 1) \quad 72k+27$	$9(8k \quad 3)$ or $72k+27$	A1
	$f(k+1) = 25f(k) \quad 9(8k \quad 3)$ or $f(k+1) = 25f(k) \quad 72k+27$ or $f(k+1) = 25(5^{2k}+3k \quad 1) \quad 72k+27$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(5^{2k}+3k \quad 1)$	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$, As the result has been shown to be true for $n=1$, then the result is true for all n ( )		A1 **cso**
			12

Question Number	Scheme	Notes	Marks
	(ii) $f(n) = 5^{2n} + 3n \quad 1$ is divisible by 9		
(ii) **Way 3**	**General Method:** Using $f(k+1) \quad mf(k)$; where m is an integer		
	$f(1) = 5^2 + 3 \quad 1 = 27$	$f(1) = 27$ is the minimum	B1
	$f(k+1) \quad mf(k) = (5^{2(k+1)} + 3(k+1) \quad 1) \quad m(5^{2k} + 3k \quad 1)$	Attempts $f(k+1) \quad mf(k)$	M1
	$f(k+1) \quad mf(k) = (25 \quad m)(5^{2k}) + 3k(1 \quad m) + (2+m)$		
	$= (25 \quad m)(5^{2k} + 3k \quad 1) \quad 9(8k \quad 3)$	$(25 \quad m)(5^{2k} + 3k \quad 1)$ or $(25 \quad m)f(k)$	A1
	or $= (25 \quad m)(5^{2k} + 3k \quad 1) \quad 72k + 27$	$9(8k \quad 3)$ or $72k + 27$	A1
	$f(k+1) = (25 \quad m)f(k) \quad 9(8k \quad 3) + mf(k)$ or $f(k+1) = (25 \quad m)f(k) \quad 72k + 27 + mf(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(5^{2k} + 3k \quad 1)$	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$, As the result has been shown to be true for $n = 1$, then the result is is true for all n ( )		A1 **cso**
(ii) **Way 4**	**General Method:** Using $f(k+1) \quad mf(k)$		
	$f(1) = 5^2 + 3 \quad 1 = 27$	$f(1) = 27$ is the minimum	B1
	$f(k+1) \quad mf(k) = (5^{2(k+1)} + 3(k+1) \quad 1) \quad m(5^{2k} + 3k \quad 1)$	Attempts $f(k+1) \quad mf(k)$	M1
	$f(k+1) \quad mf(k) = (25 \quad m)(5^{2k}) + 3k(1 \quad m) + (2+m)$		
	e.g. $m = \quad 2 \quad f(k+1) + 2f(k) = 27(5^{2k}) + 9k$	$m = \quad 2$ and $27(5^{2k})$	A1
		$m = \quad 2$ and $9k$	A1
	$f(k+1) = 27(5^{2k}) + 9k \quad 2f(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ or $(5^{2k} + 3k \quad 1)$	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$, As the result has been shown to be true for $n = 1$, then the result is true for all n ( )		A1 **cso**
	Note	Some candidates may set $f(k) = 9M$ and so may prove the following general result • $\{f(k+1) = 25f(k) \quad 9(8k \quad 3)\} \quad f(k+1) = 225M \quad 9(8k \quad 3)$ • $\{f(k+1) = 25f(k) \quad 72k + 27\} \quad f(k+1) = 225M \quad 72k + 27$	
	Question 9 Notes		
(i)	**Note**	LHS = RHS by itself is not sufficient for the 1st B1 mark in part (i).	
(i) & (ii)	**Note**	**Final A1 for parts (i) and (ii)** is dependent on all previous marks being scored in that part. It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.	
(ii)	**Note**	In part (ii), Way 4 there are many alternatives where candidates focus on isolating (5^{2k}), where is a multiple of 9. Listed below are some alternative results: $f(k+1) = 36(5^{2k}) \quad 11f(k) + 36k \quad 9$ $\quad$ $f(k+1) = 18(5^{2k}) + 7f(k) \quad 18k + 9$ $f(k+1) = 27(5^{2k}) \quad 2f(k) + 9k$ $\quad$ $f(k+1) = 9(5^{2k}) + 16f(k) \quad 45k + 18$ See the next page for how these are derived.	

	Question 9 Notes Continued			
	(ii) $f(n) = 5^{2n} + 3n \quad 1$ is divisible by 9			
9. (ii)	**The A1A1dM1 marks for Alternatives using** $f(k+1) \quad mf(k)$			
	Way 4.1	$f(k+1) = 25(5^{2k}) + 3k + 2$		
		$= 36(5^{2k}) \quad 11(5^{2k}) + 3k + 2$		
		$= 36(5^{2k}) \quad 11[(5^{2k}) + 3k \quad 1] + 36k \quad 9$	$m = \quad 11$ and $36(5^{2k})$	A1
			$m = \quad 11$ and $36k \quad 9$	A1
		$f(k+1) = 36(5^{2k}) \quad 11f(k) + 36k \quad 9$ or $f(k+1) = 36(5^{2k}) \quad 11[(5^{2k}) + 3k \quad 1] + 36k \quad 9$	as before	dM1
	Way 4.2	$f(k+1) = 25(5^{2k}) + 3k + 2$		
		$= 27(5^{2k}) \quad 2(5^{2k}) + 3k + 2$		
		$= 27(5^{2k}) \quad 2[(5^{2k}) + 3k \quad 1] + 9k$	$m = \quad 2$ and $27(5^{2k})$	A1
			$m = \quad 2$ and $9k$	A1
		$f(k+1) = 27(5^{2k}) \quad 2f(k) + 9k$ or $f(k+1) = 27(5^{2k}) \quad 2[(5^{2k}) + 3k \quad 1] + 9k$	as before	dM1
	Way 4.3	$f(k+1) = 25(5^{2k}) + 3k + 2$		
		$= 18(5^{2k}) + 7(5^{2k}) + 3k + 2$		
		$= 18(5^{2k}) + 7[(5^{2k}) + 3k \quad 1] \quad 18k + 9$	$m = 7$ and $18(5^{2k})$	A1
			$m = 7$ and $\quad 18k + 9$	A1
		$f(k+1) = 18(5^{2k}) + 7f(k) \quad 18k + 9$ or $f(k+1) = 18(5^{2k}) + 7[(5^{2k}) + 3k \quad 1] \quad 18k + 9$	as before	dM1
	Way 4.4	$f(k+1) = 25(5^{2k}) + 3k + 2$		
		$= 9(5^{2k}) + 16(5^{2k}) + 3k + 2$		
		$= 9(5^{2k}) + 16[(5^{2k}) + 3k \quad 1] \quad 45k + 18$	$m = 16$ and $9(5^{2k})$	A1
			$m = 16$ and $\quad 45k + 18$	A1
		$f(k+1) = 9(5^{2k}) + 16f(k) \quad 45k + 18$ or $f(k+1) = 9(5^{2k}) + 16[(5^{2k}) + 3k \quad 1] \quad 45k + 18$	as before	dM1

Question Number	Scheme	Notes	Marks
	$f(n) = 4^{n+2} + 5^{2n+1}$ divisible by 21		
8	$n = 1, 4^3 + 5^3 = 189 = 9 \times 21$ (Or $n = 0,\ 4^2 + 5^1 = 21$)	$f(1) = 21 \times 9$ Accept f(0) = 21 as an alternative starting point.	B1
	Assume that for $n = k$, $f(k) = (4^{k+2} + 5^{2k+1})$ is divisible by 21 for $k \in \mathbb{Z}^+$.		
	$f(k+1) - f(k) = 4^{k+3} + 5^{2k+3} - (4^{k+2} + 5^{2k+1})$	Applies $f(k+1)$ with at least 1 power correct. May be just as $f(k+1)$, or as part of an expression in $f(k+1)$ and $f(k)$.	M1
	$= 4.4^{k+2} + 25.5^{2k+1} - 4^{k+2} - 5^{2k+1}$	For a correct expression in $f(k+1)$, and possibly $f(k)$, with powers reduced to those of $f(k)$.	A1
	$= 3.4^{k+2} + 24.5^{2k+1}$		
	$= 3f(k) + 21.5^{2k+1}$ or $= 24f(k) - 21.4^{k+2}$	For one of these expression or equivalent with obvious factor of 21 in each.	A1
	$f(k+1) = 4f(k) + 21.5^{2k+1}$	Makes $f(k+1)$ the subject or gives clear reasoning of each term other than $f(k+1)$ being divisible by 21. Dependent on at least one of the previous accuracy marks being awarded.	dM1
	{ $f(k+1)$ is divisible by 21 as both $f(k)$ and 21 are both divisible by 21}		
	If the result is **true for $n = k$**, then it is now **true for $n = k + 1$**. As the result has shown to be **true for n = 1 (or 0)**, then the result is **true for all n** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for $n = 1$ stated earlier. Depends on both M's andA's, but may be scored if the B is lost as long as at least $f(1) = 189$ was reached (so e.g. if the 21×9 was not shown)	A1 cso
			(6)
ALT for first 4 marks	$n = 1, 4^3 + 5^3 = 189 = 9 \times 21$ (Or $n = 0,\ 4^2 + 5^1 = 21$)	As main scheme.	B1
	$f(k+1) - \alpha f(k) = 4^{k+3} + 5^{2k+3} - \alpha(4^{k+2} + 5^{2k+1})$	Attempts $f(k+1)$ in any equation (as main scheme).	M1
	$f(k+1) - \alpha f(k) = (4-\alpha)4^{k+2} + (25-\alpha)5^{2k+1}$	For a correct expression with any α, with powers reduced to match $f(k)$.	A1
	$f(k+1) - \alpha f(k) = (4-\alpha)(4^{k+2} + 5^{2k+1}) + 21.5^{2k+1}$ $f(k+1) - \alpha f(k) = (25-\alpha)(4^{k+2} + 5^{2k+1}) - 21.4^{k+2}$	Any suitable equation with powers sorted appropriately to match $f(k)$	A1
	NB: $\alpha = 0, \alpha = 4, \alpha = 25$ will make relevant terms disappear, but marks should be awarded accordingly.		
			Total 6

Question Number	Scheme	Notes	Marks
9(i)	$u_n = 5\times 2^{n-1} - n\times 2^n$		
	$n=1 \Rightarrow u_1 = 5\times 2^0 - 1\times 2 = 3$ (Shows the result is true for $n = 1$)		B1
	Assume true for $n = k$ so that $u_k = 5\times 2^{k-1} - k\times 2^k$		
	$u_{k+1} = 2\left(5\times 2^{k-1} - k\times 2^k\right) - 2^{k+1}$	Attempts u_{k+1} using the recurrence relationship	M1
	$= 5\times 2^k - k\times 2^{k+1} - 2^{k+1}$	Correct expanded expression	A1
	$= 5\times 2^k - (k+1)2^{k+1}$	Achieves this result with no errors	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted.		
			(5)
(ii)	$f(n) = 5^{n+2} - 4n - 9$		
	$f(1) = 125 - 4 - 9 = 112 = 16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume true for $n = k$ so that $5^{k+2} - 4k - 9$ is divisible by 16		
	$f(k+1) = 5^{k+3} - 4(k+1) - 9$	Attempts f(k + 1)	M1
	$= 5\times\left(5^{k+2} - 4k - 9\right) + \ldots$	Attempts to express in terms of f(k)	dM1
	$= 5\times\left(5^{k+2} - 4k - 9\right) + 16k + 32$	Correct expression for f(k + 1)	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		**(5)**
			Total 10

ii ALT 1	$f(1)=125-4-9=112=16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume $5^{k+2}-4k-9$ is divisible by 16		
	$f(k+1)-mf(k)=5^{k+3}-4(k+1)-9-m(5^{k+2}-4k-9)$ Attempt f(k+1) – mf(k)		M1
	$=(5-m)(5^{k+2}-4k-9)+...$	Attempts to express in terms of f(k)	dM1
	$f(k+1)=5\times(5^{k+2}-4k-9)+16k+32$	Correct expression for f(k + 1)	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		
ii ALT 2	$f(1)=125-4-9=112=16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume $5^{k+2}-4k-9$ is divisible by 16		
	$f(k+1)-f(k)=5^{k+3}-4(k+1)-9-(5^{k+2}-4k-9)$ Attempt f(k+1) – f(k)		M1
	$f(k+1)-f(k)=5\times 5^{k+2}-5^{k+2}-4k-4-9+4k+9$ $=4\times 5^{k+2}-4=4(5^{k+2}-1)$ Obtains a simplified expression for the difference **and** attempts to prove $(5^{k+2}-1)$ is divisible by 4 using induction		dM1
	Correct proof for $(5^{k+2}-1)$ being divisible by 4 and states that thus as the difference is divisible by 16, $f(k+1)$ is divisible by 16		A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1 cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		
ii ALT 3	$f(1)=125-4-9=112=16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	f(k) is divisible by 16 so set $f(k)=16\lambda$		
	$5^{k+2}=16\lambda+4k+9$		M1
	$f(k+1)=5^{k+3}-4(k+1)-9$ $=5\times 5^{k+2}-4k-13=5(16\lambda+4k+9)-4k-13$	Expresses f(k + 1) in terms of λ and k and collects terms	dM1
	$=80\lambda+16k+32$	Correct expression May have factor of 16 taken out	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		