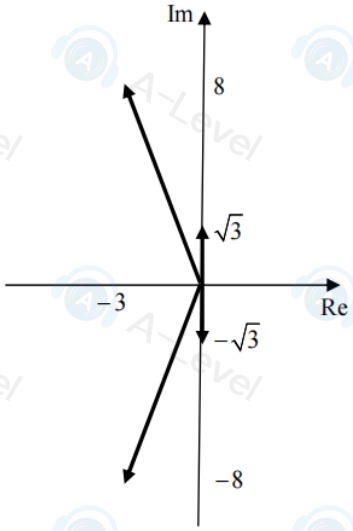
| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **1.** | $f(x)=x^4-x^3-9x^2+29x-60$ | | |
| | $1-2i$ is also a root | Seen anywhere | B1 |
| | $x^2-2x+5$ | M1: Attempt to expand $(x-(1+2i))(x-(1-2i))$ or any valid method to establish the quadratic factor | M1A1 |
| | | A1: $x^2-2x+5$ | |
| | $f(x)=(x^2-2x+5)(x^2+x-12)$ | M1: Attempt other **quadratic** factor | M1A1 |
| | | A1: $x^2+x-12$ | |
| | $x^2+x-12=(x+4)(x-3)\Rightarrow x=...$ | Attempt to solve their **other** quadratic factor. | M1 |
| | $x=-4$ and $x=3$ | Both values correct | A1 |
| | | | **(7)** |
| | | | **Total 7** |


| | | | |
|---|---|---|---|
| | | | **Total 7** |


| | | | |
|---|---|---|---|
| | **Alternative using Factor Theorem** | | |
| | $f(3)=.....$ **or** $f(-4)=.....$ | M1: Attempts f(3) **or** f(-4) | M1 |
| | $f(3)=0$ **or** $f(-4)=0$ | A1: Shows or states $f(3)=0$ **or** $f(-4)=0$ | A1 |
| | $f(3)=.....$ **and** $f(-4)=.....$ | M1: Attempts f(3) **and** f(-4) or f(3) and g(-4) where $g(x)=f(x)/(x-3)$ or f(-4) and h(3) where $h(x)=f(x)/(x+4)$ | M1 |
| | $f(3)=0$ **and** $f(-4)=0$ | A1: Shows or states $f(3)=0$ **and** $f(-4)=0$ or shows or states $f(3)=0$ and $g(-4)=0$ where $g(x)=f(x)/(x-3)$ or shows or states $f(-4)=0$ and $h(3)=0$ where $h(x)=f(x)/(x+4)$ | A1 |
| | **NB** $g(x)=x^3+2x^2-3x+20$, $h(x)=x^3-5x^2+11x-15$ | | |
| | $x=3$ **or** $x=-4$ | One of $x=3$ **or** $x=-4$ clearly stated as a **root** | M1 |
| | $x=3$ **and** $x=-4$ | Both $x=3$ **and** $x=-4$ clearly stated as **roots** | A1 |
| | $x=1-2i$ | | B1 |

| | | | |
|---|---|---|---|
| **8.** | $f(z)=z^4+6z^3+76z^2+az+b$, $a$, $b$ are real constants. $z_1=-3+8i$ is given. | | |
| (a) | $-3-8i$ | $-3-8i$ | B1 |
| | | | **(1)** |
| (b) | $z^2+6z+73$ | Attempt to expand $(z-(-3+8i))(z-(-3-8i))$ or any valid method ***to establish a quadratic factor*** eg. $z=-3\pm8i\Rightarrow z+3=\pm8i\Rightarrow z^2+6z+9=-64$ or sum of roots $-6$, product of roots 73 to give $z^2\pm(\text{sum})z+\text{product}$ | M1 |
| | | $z^2+6z+73$ | A1 |
| | $f(z)=(z^2+6z+73)(z^2+3)$ | Attempts to find the other quadratic factor. eg. using long division to get as far as $z^2+...$ or eg. $f(z)=(z^2+6z+73)(z^2+...)$ | M1 |
| | | $z^2+3$ | A1 |
| | $\{z^2+3=0\Rightarrow z=\}\ \pm\sqrt{3}i$ | **dependent on only the previous M mark** Correct method of solving the 2nd quadratic factor | dM1 |
| | | $\sqrt{3}i$ **and** $-\sqrt{3}i$ | A1 |
| | | | **(6)** |

| | | | |
|---|---|---|---|
| | | $z^2+3$ | A1 |
| | $\left\{z^2+3=0 \Rightarrow z=\right\} \pm\sqrt{3}\,\mathrm{i}$ | **dependent on only the previous M mark**<br>Correct method of solving the 2nd quadratic factor | dM1 |
| | | $\sqrt{3}\,\mathrm{i}$ **and** $-\sqrt{3}\,\mathrm{i}$ | A1 |
| | | | **(6)** |
| (c) |  | **Criteria**<br>• $-3\pm8\mathrm{i}$ plotted correctly in quadrants 2 and 3 with some evidence of symmetry<br>• Their other two ***complex roots*** (which are found from solving their 2nd quadratic in (b)) are plotted correctly with some evidence of symmetry about the *x*-axis | |
| | | Satisfies at least one of the two criteria | B1 ft |
| | | Satisfies both criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other. | B1 ft |
| | | | **(2)** |
| | | | **9** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7.** | $f(z)=z^4-6z^3+az^2-44z+b$; $a$, $b$ are real constants. $z=-1-3i$ is given. | | |
| (a) | $-1+3i$ | $-1+3i$ | B1 |
| | | | **(1)** |
| (b) | $z^2+2z+10$ | Attempt to expand $(z\pm(-1-3i))(z\pm"(-1+3i)")$ or any valid method ***to establish a quadratic factor*** e.g. $z=-1\pm3i \Rightarrow z+1=\pm3i \Rightarrow z^2+2z+1=-9$ or sum of roots $=-2$, product of roots $=10$ to give $z^2\pm$(their sum)$z\pm$(their product) | M1 |
| | | $z^2+2z+10$ | A1 |
| | $\{f(z)=\}\ (z^2+2z+10)(z^2-8z+18)$ | Attempts to find the other quadratic factor e.g. using long division to obtain $z^2+kz+...$, $k=$ value $\neq 0$ e.g. factorising/equating coefficients to obtain $f(z)=(z^2+2z+10)(z^2\pm kz\pm c)$, $k=$ value $\neq 0$, $c$ can be 0 | M1 |
| | | $z^2-8z+18$ seen in their working | A1 |
| | $\{z^2-8z+18=0\Rightarrow\}$ | | |
| | • $z=\dfrac{--8\pm\sqrt{(-8)^2-4(1)(18)}}{2(1)}$<br>• $(z-4)^2-16+18=0\Rightarrow z=...$ | **dependent on only the previous M mark** Correct method of applying the quadratic formula or completing the square **for solving their 3TQ** on their 2nd quadratic factor | dM1 |
| | $\{z=\}\ 4\pm\sqrt{2}i$ | $4+\sqrt{2}i$ **and** $4-\sqrt{2}i$ | A1 |
| | | | **(6)** |
| | | | **7** |

**Question 7 Notes**

| | | |
|---|---|---|
| **7.** (a) | **Note** | Give B1 for either $4+\sqrt{2}i$ or $4-\sqrt{2}i$ |
| (b) | **Note** | The values of the constants, i.e. $a=12$, $b=180$ do not have to be found explicitly. |
| | **Note** | You can assume $x\equiv z$ for solutions in this part |
| | **Note** | Give final dM1A1 for $z^2-8z+18=0\Rightarrow z=4+\sqrt{2}i,\ 4-\sqrt{2}i$ with no intermediate working. |
| | **Note** | They must be solving a 3TQ $"A"z^2+"B"z+"C"$ where $A$, $B$, $C$ ***are all numerical values*** $\neq 0$ for the final dM1 mark. |
| | **Note** | **Special Case:** If ***their 2nd quadratic*** factor $z^2+"B"z+"C"$ **can** be factorised then give Special Case 3rd dM1 for correct factorisation leading to $z=...$<br>Otherwise, give 3rd dM0 for applying a method of factorisation to solve their 3TQ. |
| | **Note** | **Reminder: Method mark for solving a 3TQ**<br>**Formula:** $Az^2+Bz+C=0\Rightarrow$ Attempt to use the correct formula (with values for $A$, $B$, $C$)<br>**Completing the Square:** $z^2+Bz+C=0\Rightarrow\left(z\pm\dfrac{B}{2}\right)^2\pm q\pm C=0$, $q\neq 0$, leading to $z=...$ |
| | **Note:** | **Comparing coefficients:** $f(z)=(z^2+2z+10)(z^2+\alpha z+\beta)\equiv z^4-6z^3+az^2-44z+b$<br>$z^3:\alpha+2=-6\Rightarrow\alpha=-8$; $z:2\beta+10\alpha=-44\Rightarrow 2\beta-80=-44\Rightarrow\beta=18$<br>yielding 2nd quadratic factor = $z^2-8z+18$<br>Also, constant: $10\beta=b\Rightarrow b=180$; $z^2:\beta+2\alpha+10=a\Rightarrow a=18-16+10=12$ |

| | | Question 7 Notes Continued |
|---|---|---|
| **7.** (b) | **Note:** | **<u>Long division:</u>**<br>$$\begin{array}{r} z^2 - 8z + 18 \\ z^2+2z+10 \,\big)\, \overline{z^4 - 6z^3 + az^2 - 44z + b} \\ \underline{z^4 + 2z^3 + 10z^2} \\ -8z^3 + (a-10)z^2 - 44z \\ \underline{-8z^3 - 16z^2 - 80z} \\ (a+6)z^2 + 36z + b \\ \underline{18z^2 + 36z + 180} \\ 0 \end{array}$$<br>Also, note $a=12$, $b=180$ |
| | **Note** | Ignore errors in long division for the 2nd A1 mark and/or the 3rd A1 mark. |
| | **Note** | Ignore errors in stating $a=12$, $b=180$ for the 2nd A1 mark and/or the 3rd A1 mark. |
| | **Note** | The solutions $4 \pm \sqrt{2}\,\mathrm{i}$ need to follow on from a correct $z^2 - 8z + 18$ in order to gain the final A mark. |
| | **Note** | Give final A0 for writing $\dfrac{8 \pm 2\sqrt{2}\,\mathrm{i}}{2}$ followed by either $4 \pm 2\sqrt{2}\,\mathrm{i}$ or $8 \pm \sqrt{2}\,\mathrm{i}$ |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7.** | $3x^2+px-5=0$ has roots $\alpha$, $\beta$; $p$ is a constant<br>(c) $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=2\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ | | |
| (a) (i) | $\alpha\beta=-\frac{5}{3}$ | $\alpha\beta=-\frac{5}{3}$ | B1 |
| (ii) | $\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$<br>$=\alpha\beta+2+\frac{1}{\alpha\beta}=-\frac{5}{3}+2+\frac{1}{\left(-\frac{5}{3}\right)}$ | Expands to give $\frac{1}{\alpha\beta}+1+1+\alpha\beta$; and uses their value of $\alpha\beta$ at least once in a resulting expression | M1 |
| | $=-\frac{4}{15}$ | $-\frac{4}{15}$ | A1 |
| | | | **(3)** |
| (b)(i) | $\alpha+\beta=-\frac{p}{3}$ | $\alpha+\beta=-\frac{p}{3}$ (may be recovered from (a)) | B1 |
| (ii) | $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$ | Evidence of $\frac{1}{\beta}+\frac{1}{\alpha}$ rewritten as $\frac{\alpha+\beta}{\alpha\beta}$<br>Can be implied | M1 |
| | $=-\frac{p}{3}+\frac{-\frac{p}{3}}{-\frac{5}{3}}$ or $-\frac{p}{3}+\frac{p}{5}$ or $-\frac{2p}{15}$ | $-\frac{p}{3}+\frac{-\frac{p}{3}}{-\frac{5}{3}}$ or $-\frac{p}{3}+\frac{p}{5}$ or $-\frac{2p}{15}$<br>or an equivalent fraction in terms of $p$<br>**Note:** You can apply isw | A1 |
| | | | **(3)** |
| (c) | $-\frac{2p}{15}=2\left(-\frac{4}{15}\right)\Rightarrow p=4$ | Correctly obtains $p=4$ | B1 |
| | | | **(1)** |
| (d) | $\sum=2\left(-\frac{4}{15}\right)=-\frac{8}{15}$; $\prod=-\frac{4}{15}$ | | |
| | $x^2--\frac{8}{15}x-\frac{4}{15}=0$ | Valid method for finding (their sum) **and applies** $x^2-(\text{their sum})x+\text{their product}$ (can be implied), for their numerical values of the sum and product.<br>**Note:** "$=0$" is not required for this mark<br>**Note:** E.g. Using $(\text{their sum})=\alpha+\beta=-\frac{p}{3}=-\frac{4}{3}$ is not considered a valid method for finding (their sum) | M1 |
| | $15x^2+8x-4=0$ | ***Any integer multiple*** of $15x^2+8x-4=0$, including the "$=0$" | A1 **cso** |
| | | | **(2)** |
| | | | **9** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| (a)(ii) **Way 2** | $\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$<br>$=\frac{(\alpha\beta+1)(\alpha\beta+1)}{\alpha\beta}=\frac{\left(-\frac{5}{3}+1\right)\left(-\frac{5}{3}+1\right)}{\left(-\frac{5}{3}\right)}=\frac{\frac{4}{9}}{-\frac{5}{3}}$ | Expands to give $\frac{(\alpha\beta+1)(\alpha\beta+1)}{\alpha\beta}$ and uses their value of $\alpha\beta$ at least once in a resulting expression | M1 |
| | $=-\frac{4}{15}$ | $-\frac{4}{15}$ | A1 |
| (b)(ii) **Way 2** | $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)$<br>$=\frac{(\alpha\beta+1)}{\beta}+\frac{(\alpha\beta+1)}{\alpha}=\frac{\alpha^2\beta+\alpha+\alpha\beta^2+\beta}{\alpha\beta}$ | Embedded evidence of $\frac{1}{\beta}+\frac{1}{\alpha}$ rewritten as $\frac{\alpha+\beta}{\alpha\beta}$<br>Can be implied | M1 |
| | $=\frac{\alpha\beta(\alpha+\beta)+\alpha+\beta}{\alpha\beta}$ | | |
| | $=\frac{\left(-\frac{5}{3}\right)\left(-\frac{p}{3}\right)+\left(-\frac{p}{3}\right)}{\left(-\frac{5}{3}\right)}$ or $\frac{\frac{5p}{9}-\frac{p}{3}}{-\frac{5}{3}}$ or $\frac{\frac{2p}{9}}{-\frac{5}{3}}$ or $-\frac{2p}{15}$ | Correct expression in terms of $p$<br>**Note:** You can apply isw | A1 |

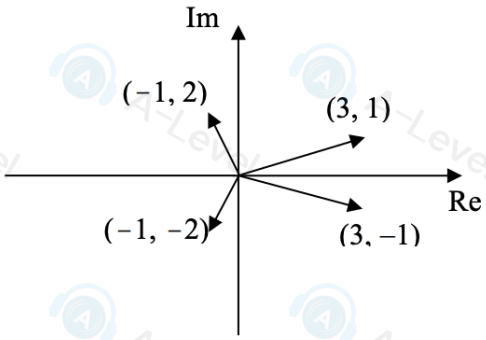
**Question 7 Notes**

| | | |
|---|---|---|
| **7.** (d) | **Note** | Valid method for finding (their sum) includes<br>• applying their $p=\ldots$ in (c) to $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=$ their $-\frac{2p}{15}$ found in (b)(ii)<br>• applying $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=2\left(\text{their } -\frac{4}{15} \text{ from (a)(ii)}\right)$ |
| | **Note** | Defining a quadratic equation $px^2+qx+r=0$ and a correct method leading to $p=15, q=8, r=-4$ without writing a final answer of $15x^2+8x-4=0$ is final M1 A0 |
| | **Note** | Give M0 for $\sum=-\frac{8}{15}$, $\prod=-\frac{4}{15}$ leading to $x^2+\frac{8}{15}-\frac{4}{15}=0$ (without recovery) |
| | **Note** | Allow M1 for $\sum=-\frac{8}{15}$, $\prod=-\frac{4}{15}$ with $x^2-(\text{sum})x+(\text{product})$ leading to $x^2+\frac{8}{15}-\frac{4}{15}=0$ |
| | **Note** | Give A1 for $15y^2+8y-4=0$ (i.e. writing their answer completely in another variable) |
| | **Note** | $\alpha,\beta=\frac{-2\pm\sqrt{19}}{3}$ and $\alpha+\frac{1}{\beta},\beta+\frac{1}{\alpha}=\frac{-4\pm2\sqrt{19}}{15}$ may be used in (d) to find the sum and product of $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$ |

| | Question 7 Notes Continued | |
|---|---|---|
| **7.** | **ALT** | For finding $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$ |
| (a) (i) | **Note** | Give B1 for $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$ and then finding $\alpha\beta = -\frac{5}{3}$ or $-\frac{60}{36}$ |
| (b) (i) | **Note** | Give B1 for $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$ and then finding $\alpha+\beta = -\frac{p}{3}$ |
| | **Note** | Allow B1 for writing $\alpha+\beta = \frac{-p+\sqrt{p^2+60}}{6} + \frac{-p-\sqrt{p^2+60}}{6}$ |
| (b)(ii) | **Note** | Allow M1 A1 for writing $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)$ as $\frac{-p+\sqrt{p^2+60}}{6} + \frac{-p-\sqrt{p^2+60}}{6} + \frac{6}{-p+\sqrt{p^2+60}} + \frac{6}{-p-\sqrt{p^2+60}}$ |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **2** | $5x^2 - 2x + 3 = 0$ | | |
| **(a)** | $\alpha + \beta = \frac{2}{5}, \quad \alpha\beta = \frac{3}{5}$ | Both correct | B1 |
| | | | **(1)** |
| **(b)(i)** | $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ | Uses a correct identity | M1 |
| | $= \left(\frac{2}{5}\right)^2 - 2\left(\frac{3}{5}\right) = -\frac{26}{25}$ | Correct value (allow $-1.04$), even after $\alpha + \beta = -\frac{2}{5}$ in (a) | A1 |
| **(ii)** | $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ | Uses a correct identity | M1 |
| | $= \left(\frac{2}{5}\right)^3 - 3\left(\frac{3}{5}\right)\left(\frac{2}{5}\right) = -\frac{82}{125}$ | Correct value (allow $-0.656$) | A1 |
| | | | **(4)** |
| **(c)** | $\text{Sum} = \alpha + \beta + \alpha^2 + \beta^2 = \frac{2}{5} - \frac{26}{25}\left(= -\frac{16}{25}\right)$ | Attempts value of sum | M1 |
| | $\text{Product} = \alpha\beta + \alpha^3 + \beta^3 + (\alpha\beta)^2 = \frac{3}{5} - \frac{82}{125} + \left(\frac{3}{5}\right)^2 \left(= \frac{38}{125}\right)$ | Attempts value of product, using the <u>correct</u> expansion of $(\alpha + \beta^2)(\beta + \alpha^2)$ | M1 |
| | $x^2 + \frac{16}{25}x + \frac{38}{125}(= 0)$ | Applies $x^2 -$ (their sum)$x +$ their product<br>Accept unsimplified versions.<br>The '$= 0$' is not required | M1 |
| | $125x^2 + 80x + 38 = 0$ | Allow any integer multiple.<br>Must be a fully correct equation, including the '$= 0$'<br>Not just $p = 125$, $q = 80$, $r = 38$ | A1 |
| | | | **(4)** |
| | | | **Total 9** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **6(a)** | $\lvert z_1+z_2\rvert\{=\lvert 3+2\mathrm{i}+2+3\mathrm{i}\rvert=\lvert 5+5\mathrm{i}\rvert\}=\sqrt{5^2+5^2}$ | Attempts the sum (allow one slip) and uses Pythagoras correctly | M1 |
| | $\sqrt{50}$ or $5\sqrt{2}$ | Either correct exact answer | A1 |
| | Answer only is no marks but working can be minimal e.g., $\lvert 5+5\mathrm{i}\rvert=5\sqrt{2}$ | | **(2)** |
| **(b)** | $\frac{z_2z_3}{z_1}=\frac{(2+3\mathrm{i})(a+b\mathrm{i})}{(3+2\mathrm{i})}=\frac{(2+3\mathrm{i})(a+b\mathrm{i})}{(3+2\mathrm{i})}\times\frac{(3-2\mathrm{i})}{(3-2\mathrm{i})}$ or $\frac{z_2}{z_1}=\frac{2+3\mathrm{i}}{3+2\mathrm{i}}\times\frac{3-2\mathrm{i}}{3-2\mathrm{i}}$ or $\frac{z_3}{z_1}=\frac{a+b\mathrm{i}}{3+2\mathrm{i}}\times\frac{3-2\mathrm{i}}{3-2\mathrm{i}}$ | Substitutes complex numbers and correct multiplier to rationalise the denominator seen or implied. See note below<br>Could use $\times\frac{-3+2\mathrm{i}}{-3+2\mathrm{i}}$ | M1 |
| | $(3+2\mathrm{i})(3-2\mathrm{i})=13$ | 13 obtained from $(3+2\mathrm{i})(3-2\mathrm{i})$<br>Could be implied. | B1 |
| | $\frac{z_2z_3}{z_1}=\frac{12a-5b}{13}+\frac{5a+12b}{13}\mathrm{i}$<br>or $\frac{1}{13}(12a-5b)+\frac{\mathrm{i}}{13}(5a+12b)$<br>or $\frac{12}{13}a-\frac{5}{13}b+\mathrm{i}\left(\frac{5}{13}a+\frac{12}{13}b\right)$ etc.<br>Condone $\frac{(12a-5b)+(5a+12b)\mathrm{i}}{13}$ | **d**M1: Attempts to simplify the numerator and collects terms to obtain $pa+qb+ra\mathrm{i}+sb\mathrm{i}$ with at least three of $p$, $q$, $r$ and $s$ non-zero. **Requires previous M mark**.<br>A1: Correct answer in any form with a single "i". Correct bracketing where needed.<br>Allow $x=\ldots$, $y=\ldots$ | **d**M1<br>A1 |
| | Note: The following marks are accessible if complex numbers are substituted in the wrong places: $z_2$ as denominator max 1010, $z_3$ as denominator max 1000 | | **(4)** |
| | Note: The following marks are accessible if complex numbers are substituted in the wrong places: $z_2$ as denominator max 1010, $z_3$ as denominator max 1000 | | **(4)** |
| **(c)** | $\frac{12a-5b}{13}=\frac{4}{13}$, $\frac{5a+12b}{13}=\frac{58}{13}$ $\Rightarrow$ $a=\ldots$, $b=\ldots$ | Equates their $x$ to $\frac{4}{13}$ and their $y$ to $\frac{58}{13}$ to obtain 2 linear equations in both $a$ and $b$ and solves to obtain values for both $a$ and $b$. | M1 |
| | No need to check values but must be some working between equations and values.<br>"$\frac{12a-5b}{13}=\frac{4}{13}$, $\frac{5a+12b}{13}=\frac{58}{13}$ $12a-5b=4$, $5a+12b=58$ $a=2$, $b=4$" is M0A0<br>Values can immediately follow if equations are produced with coefficients of $a$ or $b$ of the same magnitude | | |
| | $a=2$ and $b=4$ | Correct values for $a$ and $b$ from correct equations with working. | A1 |
| | SC: Allow access to both marks for the exact $a=-\frac{242}{169}$ and $b=\frac{716}{169}$ from using $w=\frac{z_1z_3}{z_2}=\frac{12a+5b}{13}+\frac{12b-5a}{13}\mathrm{i}$<br>There are no marks in (c) if $z_3$ was used as the denominator in (b) [leads to a = b = 0] | | **(2)** |
| **(d)** | $\arctan\left(\frac{\frac{58}{13}}{\frac{4}{13}}\right)\{=1.5019\ldots$ or $86.05\ldots^\circ\}$ or $\arctan\left(\frac{\frac{4}{13}}{\frac{58}{13}}\right)\{=0.068856\ldots$ or $3.945\ldots^\circ\}$ | Either correct arctan or $\tan^{-1}$ seen or implied by a correct 2sf value (awrt 1.5, 86, 0.069/0.068, 3.9)<br>Could use equivalent trig.<br>Note: $\tan\frac{58}{4}=-2.634$ or $0.258$ | M1 |
| | 1.502 | 1.502 only (**not** awrt)<br>Mark final answer if 1.502 is followed by e.g., $\frac{\pi}{2}-1.502=0.06880$ | A1 |
| | | | **(2)** |
| | | | **Total 10** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3** | $f(z) = z^4 + az^3 + bz^2 + cz + d$ | | |
| **(a)** | $(z =) 3 - i$ or $(z =) -1 + 2i$ | | B1 |
| | $(z =) 3 - i$ and $(z =) -1 + 2i$ | | B1 |
| | | | **(2)** |
| **(b)** |  | $3 \pm i$ correctly plotted with vectors or dots or crosses etc.<br>or<br>$-1 \pm 2i$ correctly plotted with vectors or dots or crosses etc. | B1 |
| | | All 4 correct roots correctly plotted with scaling approximately correct (e.g. (-1, 2) higher than (3, 1), etc.) There should be approximate symmetry about the real axis, but be generous | B1 |
| | | | **(2)** |
| **(c)** | $z = 3 \pm i \Rightarrow (z - (3 + i))(z - (3 - i)) = ...$<br>or<br>$z = -1 \pm 2i \Rightarrow (z - (-1 + 2i))(z - (-1 - 2i)) = ...$ | Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of $x$ (or other variable) instead of $z$ | M1 |
| | $z^2 - 6z + 10$ or $z^2 + 2z + 5$ | One correct quadratic | A1 |
| | $z^2 - 6z + 10$ and $z^2 + 2z + 5$ | Both correct | A1 |
| | $(z^2 - 6z + 10)(z^2 + 2z + 5) = ...$ | Attempts product of their two 3-term quadratic factors... no 'missing terms' in the expansion | M1 |
| | $a = -4, b = 3, c = -10, d = 50$<br>or<br>$f(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$ | All correct values or correct quartic | A1 |
| | | | **(5)** |
| | | | **Total 9** |

| | | | |
|---|---|---|---|
| **(c) Way 2** | $(z-(3+\mathrm{i}))(z-(-1\pm 2i))=\cdots$ or $(z-(3-\mathrm{i}))(z-(-1\pm 2\mathrm{i}))=\cdots$ | Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of $x$ (or other variable) instead of $z$ | M1 |
| | $z^2+z(-2+\mathrm{i})+(-1-7\mathrm{i})$ (i) or $z^2+z(-2-\mathrm{i})+(-1+7\mathrm{i})$ (ii) or $z^2+z(-2+3\mathrm{i})+(-5-5\mathrm{i})$ (iii) or $z^2+z(-2-3\mathrm{i})+(-5+5\mathrm{i})$ (iv) | One correct quadratic | A1 |
| | (i) and (ii) correct or (iii) and (iv) correct | A correct pair | A1 |
| | e,g $[z^2+z(-2+i)+(-1-7i)]\times[z^2+z(-2-\mathrm{i})+(-1+7\mathrm{i})]$ =... | Attempts product of their two 3-term quadratic factors... no 'missing terms' in the expansion | M1 |
| | $a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$ | All correct values or correct quartic | A1 |

| | | | |
|---|---|---|---|
| **(c) Way 3** | $\sum\propto=(3+\mathrm{i})+(3-\mathrm{i})+(-1-2\mathrm{i})+(-1+2\mathrm{i})$ =.. or $\alpha\beta\gamma\delta=(3+\mathrm{i})(3-\mathrm{i})(-1-2\mathrm{i})(-1+2\mathrm{i})$ =.. | Attempts one of these | M1 |
| | $a$ = - 4 or $d=50$ | | A1 |
| | Both $a$ = - 4 and $d=50$ | | A1 |
| | $\sum\alpha\beta=\ldots$ and $\sum\alpha\beta\gamma=\ldots$ | Attempts $\sum\alpha\beta$ (all 6 terms) and $\sum\alpha\beta\gamma$ (all 4 terms) | M1 |
| | $a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$ | All correct values or correct quartic | A1 |

| | | | |
|---|---|---|---|
| **(c) Way 4** | $(3+\mathrm{i})^4+a(3+\mathrm{i})^3+b(3+\mathrm{i})^2+c(3+\mathrm{i})+d=0$ (.....) + (.....) i = 0 | Substitutes one of the roots into the given quartic and fully multiplies out | M1 |
| | $(28+18a+8b+3c+d)+\mathrm{i}(96+26a+6b+c)$ or $(-7+11a-3b-c+d)+\mathrm{i}(-24+2a+4b-2c)$ | One correct expansion | A1 |
| | $(28+18a+8b+3c+d)+\mathrm{i}(96+26a+6b+c)$ and $(-7+11a-3b-c+d)+\mathrm{i}(-24+2a+4b-2c)$ | Obtains a second correct expansion using another root. | A1 |
| | $(28+18a+8b+3c+d)=0$, etc leading to $a=$ , $b=$ , $c=$ , $d=$ | Solves 4 simultaneous equation to find values of $a$, $b$, $c$ and $d$ | M1 |
| | $a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$ | All correct values or correct quartic | A1 |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3.(a)** | $\alpha+\beta=\frac{7}{2}$ **and** $\alpha\beta=2$ | Allow $\frac{4}{2}$ for 2 | B1 |
| | $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$<br>$=\left(\frac{7}{2}\right)^2-2(2)=\frac{33}{4}$ | M1: Uses $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$<br>A1: $\frac{33}{4}$ or $8\frac{1}{4}$ or 8.25 | M1 A1 |
| | | | (3) |
| **(b)** | Sum of roots is<br>$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{33/4}{2}=\frac{33}{8}$<br>and product of roots is $\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=1$ | M1: Attempts sum **or** product of new roots correctly (may be implied)<br>A1: Sum = $\frac{33}{8}$ and product = 1 | M1 A1 |
| | $x^2-\frac{33}{8}x+1=0\therefore 8x^2-33x+8=0$ | $8x^2-33x+8=0$ or any integer multiple including the "= 0" | A1 |
| | | | (3) |
| | | | **[6 marks]** |

| | **Alternative – finds roots explicitly:** | | |
|---|---|---|---|
| **(a)** | $\alpha,\beta=\frac{1}{4}\left(7\pm\sqrt{17}\right)$ | Correct exact roots including $\sqrt{17}$ | B1 |
| | $\alpha^2+\beta^2=2\left(\frac{49}{16}\right)+2\frac{17}{16}=2\times\frac{66}{16}=\frac{33}{4}$ | M1: Squares and adds their roots<br>A1: cao $\frac{33}{4}$ or $8\frac{1}{4}$ or 8.25 | M1 A1 |
| | | | (3) |
| **(b)** | $\left(x-\frac{7+\sqrt{17}}{7-\sqrt{17}}\right)\left(x-\frac{7-\sqrt{17}}{7+\sqrt{17}}\right)=\ldots$ | Uses $\left(x-\frac{\alpha}{\beta}\right)\left(x-\frac{\beta}{\alpha}\right)$ with numerical $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ and attempts to expand. There are no marks until numerical values are used. | M1 |
| | $=x^2-\frac{33}{8}x+1$ | | A1 |
| | $8x^2-33x+8=0$ | This answer with no errors or any integer multiple including the "= 0" | A1cso |
| | | | (3) |
| | **Note:**<br>**Roots of the form $\frac{1}{k}\left(7\pm\sqrt{17}\right)$, $k\neq 4$ will give a correct answer – in this case lose the final mark as not cso.** | | |