Question Number	Scheme	Notes	Marks
4. (a) **Way 1**	$\left\{x=4t,\ y=\frac{4}{t}\ \Rightarrow\right\}\ 3\left(\frac{4}{t}\right)\quad 2(4t)=10$	Substitutes $x=4t$ **and** $y=\frac{4}{t}$ into the printed equation to obtain an equation in t only	M1
	$8t^2+10t\quad 12=0$ or $4t^2+5t\quad 6=0$ **(*can be implied*)**	A correct 3 term quadratic **Note:** E.g. $12\quad 8t^2=10t$, $8t^2+10t\quad 12\{=0\}$ or $8t^2+10t=12$ are acceptable for this mark	A1
	$(8t\quad 6)(t+2)=0\qquad t=...$ or $(4t\quad 3)(2t+4)=0\qquad t=...$ or $(4t\quad 3)(t+2)=0\qquad t=...$	**dependent on the previous M mark** Correct method (e.g. factorising, completing the square or applying the quadratic formula) of solving a 3TQ to find $t=...$	dM1
	• $x=4\left(\frac{3}{4}\right)=3$ **and** $y=\frac{4}{\left(\frac{3}{4}\right)}=\frac{16}{3}$ • $x=4(\ \ 2)=\ \ 8$ **and** $y=\frac{4}{(\ \ 2)}=\ \ 2$	**dependent on both the previous M marks** Correct substitution at least one of their values for t into the given parametric equations **and** obtains ***two sets*** of corresponding values for $x=...$ **and** $y=...$	ddM1
	$A\left(3,\frac{16}{3}\right)$, $B(\ \ 8,\ \ 2)$ **or** A: $x=3$, $y=\frac{16}{3}$ **and** B: $x=\ \ 8$, $y=\ \ 2$	Identifies the correct coordinates for A and B	A1 **cao**
			(5)
(a) **Way 2**	$x\left(\frac{10+2x}{3}\right)=16$	$\left(\frac{3y\quad 10}{2}\right)y=16$	**Either** substitutes their rearranged $3y\quad 2x=10$ into $xy=k$ **or** substitutes either $y=\frac{k}{x}$ or $x=\frac{k}{y}$, $k\quad 0$, into $3y\quad 2x=10$ to form an equation in either x only or y only	M1
	$3\left(\frac{16}{x}\right)\quad 2x=10$	$3y\quad 2\left(\frac{16}{y}\right)=10$		
	$2x^2+10x\quad 48=0$ or $x^2+5x\quad 24=0$ or $\frac{2}{3}x^2+\frac{10}{3}x\quad 16=0$ or $\frac{3}{2}y^2\quad 5y\quad 16=0$ or $3y^2\quad 10y\quad 32=0$ **(*can be implied*)**	A correct 3 term quadratic **Note:** $10x+2x^2=48$, $3y^2\quad 10y=32$ or $x^2+5x\quad 24\{=0\}$ are acceptable for this mark	A1
	e.g. $(2x+16)(x\quad 3)=0\qquad x=...$ or $(x+8)(x\quad 3)=0\qquad x=...$ or $(3y\quad 16)(y+2)=0\qquad y=...$	**dependent on the previous M mark** Correct method (e.g. factorising, completing the square or applying the quadratic formula) of solving a 3TQ to find either $x=...$ or $y=...$	dM1
	E.g. $x=3\quad y=\frac{16}{3}$ $x=\ \ 8\quad y=\frac{16}{8}=\ \ 2$	**dependent on both the previous M marks.** Correct substitution of at least one of their values for x or y into either $3y\quad 2x=10$ **or** their rearranged $3y\quad 2x=10$ or $y=\frac{k}{x}$ or $x=\frac{k}{y}$, $k\quad 0$, **and** obtains ***two sets*** of corresponding values for $x=...$ **and** $y=...$	ddM1
	$A\left(3,\frac{16}{3}\right)$, $B(\ \ 8,\ \ 2)$ **or** A: $x=3$, $y=\frac{16}{3}$ **and** B: $x=\ \ 8$, $y=\ \ 2$	Identifies the correct coordinates for A and B	A1 **cao**
			(5)
(b)	$\left(\frac{3+(\ \ 8)}{2},\frac{\frac{16}{3}+(\ \ 2)}{2}\right);=\left(\ \ \frac{5}{2},\frac{5}{3}\right)$	Uses their (x_1,y_1) and (x_2,y_2) from part (a) to apply $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ o.e.	M1;
		Correct answer	A1
			(2)
			7

		Question 4 Notes
4. (a)	**SC**	If the two previous M marks have been gained then award Special Case ddM1 for finding their correct points by writing either $x=3,\ y=\frac{16}{3}$ **or** $x=\ \ 8,\ y=\ \ 2$ **or** $\left(3,\frac{16}{3}\right)$ **or** $(\ \ 8,\ \ 2)$
	Note	A decimal answer e.g. $A(3,5.33),\ B(\ \ 8,\ \ 2)$ (without a correct exact answer) is 2nd A0
	Note	**Writing coordinates the wrong way round** E.g. writing $x=3,\ y=\frac{16}{3}$ **and** $x=\ \ 8,\ y=\ \ 2$ followed by $A\left(\frac{16}{3},3\right),\ B(\ \ 8,\ \ 2)$ is 2nd A0
	Note	Imply the dM1 mark for ***writing down*** the ***correct*** roots for ***their*** quadratic equation. E.g. • $2x^2+10x\ \ \ 48=0$ or $x^2+5x\ \ \ 24=0$ or $\frac{2}{3}x^2+\frac{10}{3}x=16\rightarrow x=3,\ \ \ 8$ • $\frac{3}{2}y^2\ \ \ 5y\ \ \ 16=0$ or $3y^2\ \ \ 10y\ \ \ 32=0\rightarrow y=\frac{16}{3},\ \ \ 2$ • $8t^2+10t=12$ or $4t^2+5t\ \ \ 6=0\rightarrow t=\frac{3}{4},\ \ \ 2$
	Note	For example, give dM0 for • $8t^2+10t=12$ or $4t^2+5t\ \ \ 6=0\rightarrow t=\frac{1}{4},\ \ \ 2$ [incorrect solution] with no intermediate working.
	Note	You can also imply the 1st A1 dM1 marks for either • $x\left(\frac{10+2x}{3}\right)=16$ or $3\left(\frac{16}{x}\right)\ \ \ 2x=10\rightarrow x=3,\ \ \ 8$ • $\left(\frac{3y\ \ \ 10}{2}\right)y=16$ or $3y\ \ \ 2\left(\frac{16}{y}\right)=10\rightarrow y=\frac{16}{3},\ \ \ 2$ • $3\left(\frac{4}{t}\right)\ \ \ 2(4t)=10\rightarrow x=3,\ \ \ 8$ • $3\left(\frac{4}{t}\right)\ \ \ 2(4t)=10\rightarrow y=\frac{16}{3},\ \ \ 2$ with no intermediate working.
	Note	You can imply the 1st A1 dM1 ddM1 marks for either • $x\left(\frac{10+2x}{3}\right)=16$ or $3\left(\frac{16}{x}\right)\ \ \ 2x=10\rightarrow x=3,\ \ \ 8$ **and** $y=\frac{16}{3},\ \ \ 2$ • $3\left(\frac{4}{t}\right)\ \ \ 2(4t)=10\rightarrow x=3,\ \ \ 8$ **and** $y=\frac{16}{3},\ \ \ 2$ with no intermediate working. You can then imply the final A1 mark if they correctly identify the correct pairs of values or coordinates which relate to the point *A* and the point *B*.
	Note	Give 2nd A0 for a final answer of **both** $A\left(3,\frac{16}{3}\right),\ B(\ \ 8,\ \ 2)$ **and** $A(\ \ 8,\ \ 2),\ B\left(3,\frac{16}{3}\right)$,
(b)	**Note**	A decimal answer e.g. $(\ \ 2.5,1.67)$ (without a correct exact answer) is A0
	Note	Allow A1 for $\left(\ \ \frac{5}{2},\frac{10}{6}\right)$ or $\left(\ \ 2\frac{1}{2},\ \ 1\frac{2}{3}\right)$ or exact equivalent.

	interval bisection. There are no marks if it is a clear attempt at interpolation.		
(c)(i)	$f'(x)=\frac{3}{2}x^{\frac{1}{2}}+1$	Correct differentiation. Any correct equivalent e.g., $1.5\sqrt{x}+1$	B1
(ii)	$\alpha \approx 1.375-\frac{1.375^{\frac{3}{2}}+1.375-3}{"\frac{3}{2}\times 1.375^{\frac{1}{2}}+1"}=...$ $\left\{=1.375-\frac{-0.01266958256...}{2.75890591...}=1.375+0.004592248875...\right.$ $\left.=1.379592249...\right\}$ $\left\{\text{exact values}: \frac{11}{8}-\frac{11\sqrt{22}-52}{32}\div\frac{8+3\sqrt{22}}{8}\right\}$	Correctly applies the Newton-Raphson formula with 1.375 & their $f'(x)$ **and** obtains a value. Some working must be seen unless approx. root is seen correct to 6 d.p. accuracy (1.379592) or better. Allow "$...=1.375-\frac{f(1.375)}{f'(1.375)}$" followed by value but formula must be **fully** substituted if just followed by value unless "x_0" defined	M1
	awrt 1.380 or "1.38" (Ignore further iterations)	No clearly incorrect work.	A1
	NB Actual root is 1.379589808. Answer only is no marks.		**(3)**
(d)	e.g., $\frac{\alpha-1.25}{1.5-\alpha}=\frac{0.3524575141...}{0.3371173071...}$ or e.g., $\frac{1.5-\alpha}{0.337...}=\frac{1.5-1.25}{0.337...+0.352...}$	Forms an equation in e.g., α with their f(1.25) and f(1.5) allowing for sign errors only but must be using differences. Allow use of "f(1.25)" and "f(1.5)"- could recover sign error	M1
	$\alpha=1.377780737...=1.378$	**d**M1: Solves $\Rightarrow$ value **Requires previous M mark.** A1: awrt 1.378	**d**M1 A1
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		**(3)**
Alt (Equation of line methods)	or $y-(-0.3524...[\text{or } 0.3371...])=\frac{0.3371...-(-0.3524...)}{1.5-1.25}(x-1.25[\text{or } 1.5])$ or $-0.3524...[\text{or } 0.3371...]=\frac{0.3371...-(-0.3524...)}{1.25[\text{or } 1.5]}+c\Rightarrow c=...$		M1

	interval bisection. There are no marks if it is a clear attempt at interpolation.		
(c)(i)	$f'(x)=\frac{3}{2}x^{\frac{1}{2}}+1$	Correct differentiation. Any correct equivalent e.g., $1.5\sqrt{x}+1$	B1
(ii)	$\alpha \approx 1.375-\frac{1.375^{\frac{3}{2}}+1.375-3}{"\frac{3}{2}\times 1.375^{\frac{1}{2}}+1"}=...$ $\left\{=1.375-\frac{-0.01266958256...}{2.75890591...}=1.375+0.004592248875...\right.$ $\left.=1.379592249...\right\}$ $\left\{\text{exact values}: \frac{11}{8}-\frac{11\sqrt{22}-52}{32}\div\frac{8+3\sqrt{22}}{8}\right\}$	Correctly applies the Newton-Raphson formula with 1.375 & their $f'(x)$ **and** obtains a value. Some working must be seen unless approx. root is seen correct to 6 d.p. accuracy (1.379592) or better. Allow "$...=1.375-\frac{f(1.375)}{f'(1.375)}$" followed by value but formula must be **fully** substituted if just followed by value unless "x_0" defined	M1
	awrt 1.380 or "1.38" (Ignore further iterations)	No clearly incorrect work.	A1
	NB Actual root is 1.379589808. Answer only is no marks.		**(3)**
(d)	e.g., $\frac{\alpha-1.25}{1.5-\alpha}=\frac{0.3524575141...}{0.3371173071...}$ or e.g., $\frac{1.5-\alpha}{0.337...}=\frac{1.5-1.25}{0.337...+0.352...}$	Forms an equation in e.g., α with their f(1.25) and f(1.5) allowing for sign errors only but must be using differences. Allow use of "f(1.25)" and "f(1.5)"- could recover sign error	M1
	$\alpha=1.377780737...=1.378$	**d**M1: Solves $\Rightarrow$ value **Requires previous M mark.** A1: awrt 1.378	**d**M1 A1
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		**(3)**
Alt (Equation of line methods)	or $y-(-0.3524...[\text{or } 0.3371...])=\frac{0.3371...-(-0.3524...)}{1.5-1.25}(x-1.25[\text{or } 1.5])$ or $-0.3524...[\text{or } 0.3371...]=\frac{0.3371...-(-0.3524...)}{1.25[\text{or } 1.5]}+c\Rightarrow c=...$		M1

	$\left\{=1.375-\dfrac{-0.01266958256...}{2.75890591...}=1.375+0.004592248875...\right\}$ $=1.379592249...$ $\left\{\text{exact values}: \dfrac{11}{8}-\dfrac{11\sqrt{22}-52}{32}\div\dfrac{8+3\sqrt{22}}{8}\right\}$	working must be seen unless approx. root is seen correct to 6 d.p. accuracy (1.379592) or better. Allow "$...=1.375-\dfrac{f(1.375)}{f'(1.375)}$" followed by value but formula must be **fully** substituted if just followed by value unless "x_0" defined	M1
	awrt 1.380 or "1.38" (Ignore further iterations)	No clearly incorrect work.	A1
	NB Actual root is 1.379589808. Answer only is no marks.		**(3)**
(d)	e.g., $\dfrac{\alpha-1.25}{1.5-\alpha}=\dfrac{0.3524575141...}{0.3371173071...}$ or e.g., $\dfrac{1.5-\alpha}{0.337...}=\dfrac{1.5-1.25}{0.337...+0.352...}$	Forms an equation in e.g., α with their f(1.25) and f(1.5) allowing for sign errors only but must be using differences. Allow use of "f(1.25)" and "f(1.5)"- could recover sign error	M1
	$\alpha=1.377780737...=1.378$	**d**M1: Solves $\Rightarrow$ value **Requires previous M mark.** A1: awrt 1.378	**d**M1 A1
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		**(3)**
Alt (Equation of line methods)	or $y-(-0.3524...[\text{or } 0.3371...])=\dfrac{0.3371...-(-0.3524...)}{1.5-1.25}(x-1.25[\text{or} 1.5])$ or $-0.3524...[\text{or } 0.3371...]=\dfrac{0.3371...-(-0.3524...)}{1.5-1.25}(1.25[\text{or} 1.5])+c\Rightarrow c=...$ A full method to determine the equation of the line using their f(1.25) and f(1.5) allowing for sign errors only (but allow subsequent errors finding c if $y=mx+c$ used)		M1
	$\{\Rightarrow y=2.758...x-3.800...\}$ $\alpha=1.377780737...=1.378$	**d**M1: Puts $y=0$ and solves $\Rightarrow$ value **Requires previous M mark.** A1: awrt 1.378	**d**M1 A1 **(3)**
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		**Total 11**

Question	Scheme	Marks
7(a)	$\frac{dy}{dx} = -\frac{36}{x^2}$ oe	B1
	$m_t = -\frac{36}{4^2} \Rightarrow m_n = \frac{16}{36} = \frac{4}{9}$	M1
	Normal is $y-9=\frac{4}{9}(x-4)$	M1
	$\Rightarrow 9y-81=4x-16 \Rightarrow 4x-9y+65=0$ *	A1*
		(4)
(b)	Normal meets H again when $4x-9\times\frac{36}{x}+65=0$ or $4\times\frac{36}{y}-9y+65=0$	M1
	$\Rightarrow 4x^2+65x-324=0 \Rightarrow x=...$ or $9y^2-65y-144=0 \Rightarrow y=...$	dM1
	$\Rightarrow Q=\left(-\frac{81}{4},-\frac{16}{9}\right)$	A1
	At $x=-\frac{81}{4}$, $\frac{dy}{dx}=-\frac{36}{\left(-\frac{81}{4}\right)^2}=...$ so tangent is $y-\left(-\frac{16}{9}\right)=-\frac{64}{729}\left(x-\left(-\frac{81}{4}\right)\right)$	M1
	$y=-\frac{64}{729}x-\frac{32}{9}$	A1(5)
		(9 marks)

Notes:

(a)

B1: Correct expression for $\frac{dy}{dx}$, or any equivalent correct expression including it, such as $x\frac{dy}{dx}+y=0$ or $\frac{dy}{dx}=-\frac{1}{t^2}$

M1: Attempts negative reciprocal gradient at the point P

M1: Uses their normal (changed from tangent) gradient and $P(4,9)$ to find the equation of the normal. Look for $y-9="m_n"(x-4)$ Working must be shown for their constant if $y=mx+c$ is used as this is a "show that" question.

A1*: Correct equation achieved from correct working with intermediate step.

(b)

M1: Substitutes hyperbola equation into the **given** normal to obtain an equation in one variable. Other valid means of obtaining an equation in a single variable are acceptable.

dM1: Gathers terms and solves the 3 term quadratic to find a value $\neq 4$ for x or $\neq 9$ for y. Solution by calculator allowed if correct roots (or values $\neq 4$ for x or $\neq 9$ for y) are shown

A1: Correct coordinates of intersection.

M1: Uses their x value to find the gradient at Q and then uses the intersection point with their gradient to form the equation of the line.

A1: Correct equation.

Question Number	Scheme	Notes	Marks
10.	$H: xy = 144;\ \ P\left(12p, \dfrac{12}{p}\right),\ p \neq 0,$ lies on H. Normal to H at P crosses positive x-axis at Q and negative y-axis at R		
(a)	$y = \dfrac{144}{x} = 144x^{-1} \Rightarrow \dfrac{dy}{dx} = -144x^{-2}$ or $-\dfrac{144}{x^2}$	$\dfrac{dy}{dx} = \pm kx^{-2};\ k \neq 0$	M1
	$xy = 144 \ \Rightarrow\ x\dfrac{dy}{dx} + y = 0$	Uses product rule to give $\pm x\dfrac{dy}{dx} \pm y$	
	$x = 12t,\ y = \dfrac{12}{t} \ \Rightarrow\ \dfrac{dy}{dx} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = -\left(\dfrac{12}{t^2}\right)\left(\dfrac{1}{12}\right)$	their $\dfrac{dy}{dt} \times \dfrac{1}{\text{their } \dfrac{dy}{dt}}$; **Condone** $t \equiv p$	
	So at P, $m_T = -\dfrac{1}{p^2}$	Correct calculus work leading to $m_T = -\dfrac{1}{p^2}$	A1
	So, $m_N = p^2$	Applies $m_N = \dfrac{-1}{m_T}$, where m_T is found using calculus	M1
	• $y - \dfrac{12}{p} = "p^2"(x - 12p)$ **or** • $\dfrac{12}{p} = "p^2"(12p) + c \ \Rightarrow\ y = "p^2"x + \text{their } c$	Correct straight line method for an equation of a normal where $m_N\ (\neq m_T)$ is found by using calculus.	M1
	Correct algebra leading to $y = p^2x + \dfrac{12}{p} - 12p^3$ *	Correct solution only	A1 *
	Note: m_N must be a function of p for the 2nd M1 and 3rd M1 mark		**(5)**
(b)	$y = 0 \ \Rightarrow\ x_Q = 12p - \dfrac{12}{p^3}$	Puts $y = 0$ and finds x **or** puts $x = 0$ and finds y	M1
	$x = 0 \Rightarrow y_R = \dfrac{12}{p} - 12p^3$	At least one of x_Q or y_R correct, o.e.	A1
	$\left(12p - \dfrac{12}{p^3}, 0\right)$ and $\left(0, \dfrac{12}{p} - 12p^3\right)$	Both sets of coordinates correct. {Ignore labelling of coordinates}	A1
			(3)
(c)	Area $OQR = \dfrac{1}{2}\left(12p - \dfrac{12}{p^3}\right)\left\|\left(\dfrac{12}{p} - 12p^3\right)\right| = 512$	$\dfrac{1}{2} \times (\pm \text{their } x_Q)(\pm \text{their } y_R) = 512$	M1
		Correct equation which can be un-simplified or simplified	A1
	$144p^4 - 1312 + \dfrac{144}{p^4} = 0$		
	$144p^8 - 1312p^4 + 144 = 0$ $\{\Rightarrow 9p^8 - 82p^4 + 9 = 0\}$	Correct 3 term quadratic in p^4 **Note:** $144p^8 + 144 = 1312p^4$ is acceptable for this mark	A1
	$(9p^4 - 1)(p^4 - 9) = 0 \ \Rightarrow\ p^4 = \ldots$	**dependent on the previous M mark** Uses a 3TQ in p^4 (or an implied 3TQ in p^4) to find at least one value of $p^4 = \ldots$	dM1
	$p = \sqrt{3}$ and $p = -\dfrac{1}{\sqrt{3}}$	Obtains both $p = \sqrt{3}$ and $p = -\dfrac{1}{\sqrt{3}}$ ***only*** **Note: Allow** $p = -\dfrac{\sqrt{3}}{3}$ in place of $p = -\dfrac{1}{\sqrt{3}}$	A1
			(5)
			13

Question Number	Scheme	Notes	Marks
10. (c)	Area $OQR = \frac{1}{2}\left(12p-\frac{12}{p^3}\right)\left\|\left(\frac{12}{p}-12p^3\right)\right\| = 512$	$\frac{1}{2}\times(\pm \text{ their } x_Q)(\pm \text{ their } y_R) = 512$	M1
		Correct equation which can be un-simplified or simplified	A1
	$144\left(p-\frac{1}{p^3}\right)\left(p^3-\frac{1}{p}\right)=1024 \Rightarrow p^4-2+\frac{1}{p^4}=\frac{1024}{144}$		
	$\left(p^2-\frac{1}{p^2}\right)^2=\frac{64}{9} \Rightarrow p^2-\frac{1}{p^2}=\pm\frac{8}{3}$		
	$3p^4-8p^2-3=0$ **and** $3p^4+8p^2-3=0$	Both correct 3 term quadratics in p^2 **Note:** Both $p^4-1=\frac{8}{3}p^2$ and $3p^4+8p^2=3$ is acceptable for this mark	A1
	$(3p^2+1)(p^2-3)=0 \Rightarrow p^2=\ldots$ **or** $(3p^2-1)(p^2+3)=0 \Rightarrow p^2=\ldots$	**dependent on the previous M mark** Uses a 3TQ in p^2 (or an implied 3TQ in p^2) to find at least one value of $p^2=\ldots$	dM1
	$p=\sqrt{3}$ and $p=-\frac{1}{\sqrt{3}}$	Obtains both $p=\sqrt{3}$ and $p=-\frac{1}{\sqrt{3}}$ ***only***	A1
			(5)

Question 10 Notes

10. (a)	**Note**	Allow $y=p^2x-12p^3+\frac{12}{p}$ {order of terms interchanged in $y=\ldots$} for final A1
(b)	**Note**	For the accuracy marks in part (b) allow equivalents such as • $x=12p-\frac{12}{p^3}$ or $x=\frac{12p^4-12}{p^3}$ or $x=\frac{12(p^2-1)(p^2+1)}{p^3}$ • $y=\frac{12}{p}-12p^3$ or $y=\frac{12-12p^4}{p}$
(c)	**Note**	Give 1st M1, 1st A1 for • $\frac{1}{2}\left(12p-\frac{12}{p^3}\right)\left\|\left(\frac{12}{p}-12p^3\right)\right\| = 512$ {correct use of modulus} • $\frac{1}{2}\left(12p-\frac{12}{p^3}\right)\left(12p^3-\frac{12}{p}\right) = 512$ {modulus has been applied here} • $-\frac{1}{2}\left(12p-\frac{12}{p^3}\right)\left(\frac{12}{p}-12p^3\right) = 512$ {modulus has been applied here}
	Note	Give 1st M1, 1st A0 for $\frac{1}{2}\left(12p-\frac{12}{p^3}\right)\left(\frac{12}{p}-12p^3\right) = 512$ {modulus has not been applied on y_R}
	Note	Writing a correct $144p^4-1312+\frac{144}{p^4}=0$ o.e. followed by a correct e.g. $p^4=9$ with no intermediate working is 2nd A0, 2nd M1
	Note	Writing a correct $144p^4-1312+\frac{144}{p^4}=0$ o.e. followed by $p^4=9$ **and** $p^4=\frac{1}{9}$ with no intermediate working is 2nd A1 (implied), 2nd M1

Question Number	Scheme	Notes	Marks
9.	$C: y^2 = 4ax$; $P(ap^2, 2ap)$ lies on C; circle: $(x-10a)^2 + y^2 = \frac{9}{4}a^2$		
(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}} = \frac{\sqrt{a}}{\sqrt{x}}$	$\frac{dy}{dx} = \pm k x^{-\frac{1}{2}}$; $k \neq 0$	M1
	$2y\frac{dy}{dx} = 4a$	$ky\frac{dy}{dx} = c$; $k, c \neq 0$	
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 2a\left(\frac{1}{2ap}\right)$	their $\frac{dy}{dt} \times \frac{1}{\text{their}\frac{dx}{dt}}$; Condone $t \equiv p$	
	{At P, $x = ap^2$, $y = 2ap \Rightarrow$} $\frac{dy}{dx} = \frac{1}{p}$	Correct calculus work leading to $m_T = \frac{1}{p}$	A1
	So, at P, $m_N = -p$	Applies $m_N = \frac{-1}{m_T}$, to find m_N in terms of p, where m_T is found using calculus. Can be implied by later working.	M1
	either $y - 2ap = -p(x - ap^2)$ **or** $y - 0 = -p(x - 10a)$	Correct straight line method for an equation of a normal, where m_N $(\neq m_T)$ is found using calculus.	M1
	$0 - 2ap = -p(10a - ap^2)$ $\quad 2ap - 0 = -p(ap^2 - 10a)$ $\Rightarrow p = \ldots \Rightarrow x = \ldots$ or $y = \ldots$ **or** $2ap - 0 = -p(x - 10a) \Rightarrow x = \ldots$	**dependent on the previous M mark** Complete method to find either the x or y coordinate of P	dM1
	either $x = 8a$, $y = 4\sqrt{2}\,a$ **or** $P(8a, 4\sqrt{2}\,a)$	**Either** $x = 8a$ **or** $y = 4\sqrt{2}\,a$ **or** $y =$ awrt $5.66a$	A1
		$P(8a, 4\sqrt{2}\,a)$ or both $x = 8a$ and $y = 4\sqrt{2}\,a$	A1
	Note: $p = 2\sqrt{2}$ or $\sqrt{8}$. **Note:** Ignore the additional solution $P(8a, -4\sqrt{2}\,a)$		**(7)**
(b) **Way 1**	Area $SBP = \frac{1}{2}(10a - a)(4\sqrt{2}a)$	$\frac{1}{2}(10a - a)$(their y_P from (a))	M1
	$= 18\sqrt{2}\,a^2$	$18\sqrt{2}\,a^2$	A1
			(2)
(c) **Way 1**	$PB = \sqrt{(10a - "8a")^2 + ("4\sqrt{2}a")^2}$ $\{= 6a\}$	Complete Pythagoras method for finding length PB	M1
	$PR = 6a - 1.5a$	**dependent on the previous M mark** $PR =$ "their $6a$" $- 1.5a$	dM1
	$PR = 4.5a$	$PR = 4.5a$	A1
			(3)
(c) **Way 2**	$p = 2\sqrt{2} \Rightarrow l: y = -2\sqrt{2}x + 20\sqrt{2}a$ $(x-10a)^2 + (-2\sqrt{2}x + 20\sqrt{2}a)^2 = \frac{9}{4}a^2$ $\Rightarrow 36x^2 - 720ax + 3591a^2 = 0$ $\Rightarrow 9(2x - 21a)(2x - 19a) = 0 \Rightarrow x = \ldots$	Substitutes their equation of l into the circle equation followed by a correct method for solving their 3TQ to give $x = \ldots$ or $y = \ldots$	M1
	$\Rightarrow R(9.5a, \sqrt{2}\,a)$		
	$PR = \sqrt{(9.5a - 8a)^2 + (\sqrt{2}a - 4\sqrt{2}\,a)^2}$	**dependent on the previous M mark** Complete applied Pythagoras method for finding the distance between their P and their R	dM1
	$PR = 4.5a$	$PR = 4.5a$	A1
			(3)

Question Number	Scheme	Notes	Marks
9. (b) **Way 2**	Area $SBP = \frac{1}{2}\begin{vmatrix} a & 10a & "8a" & a \\ 0 & 0 & "4\sqrt{2}a" & 0 \end{vmatrix}$ $= \frac{1}{2}\lvert 0 - 0 + 40\sqrt{2}\,a^2 - 0 + 0 - 4\sqrt{2}\,a^2 \rvert$	Complete applied method for finding area SBP using $S(a, 0)$, $B(10a, 0)$ and their P from (a)	M1
	$= 18\sqrt{2}\,a^2$	$18\sqrt{2}\,a^2$	A1
			(2)
9. (c) **Way 3**	$x_R = 10a - 1.5\cos\left(\tan^{-1}\left(\frac{"4\sqrt{2}\,a"}{10a - "8a"}\right)\right)$ $y_R = 1.5\sin\left(\tan^{-1}\left(\frac{"4\sqrt{2}\,a"}{10a - "8a"}\right)\right)$	Uses their P from (a) in a correct method for writing down either x_R or y_R	M1
	$\Rightarrow R(9.5a, \sqrt{2}\,a)$		
	$PR = \sqrt{(9.5a - 8a)^2 + (\sqrt{2}a - 4\sqrt{2}\,a)^2}$	**dependent on the previous M mark** Complete applied Pythagoras method for finding the distance between their P and their R	dM1
	$PR = 4.5a$	$PR = 4.5a$	A1
			(3)

Question 9 Notes

9. (a)	**Note**	Allow 1st M1 1st A1 (sufficient use of calculus) for $\{m_T =\}\ \frac{4a}{2y}$ which leads to $\{m_T =\}\ \frac{1}{p}$		
	Note	Allow 1st M1 1st A1 (sufficient use of calculus) for $\{m_T =\}\ \sqrt{\frac{a}{x}}$ which leads to $\{m_T =\}\ \frac{1}{p}$		
	Note	Give 3rd M1 for either • $2ap = "(-p)"(ap^2) + c \Rightarrow y = "(-p)"x +$ their c **or** • $0 = "(-p)"(10a) + c \Rightarrow y = "(-p)"x +$ their c		
	Note	Writing coordinates the wrong way around E.g. finding $x = 8a$, $y = 4\sqrt{2}\,a$ followed by $(4\sqrt{2}\,a, 8a)$ is final A0		
	Note	Give final A0 for $(8a, 5.65685...a)$ without reference to $y = 4\sqrt{2}\,a$ or $2\sqrt{8}\,a$		
	Note	Accept $y_P = 2\sqrt{8}\,a$ written in place of $y_P = 4\sqrt{2}\,a$ for the final A1 A1 marks		
	Note	**Special Case** If they write down either $\frac{dy}{dx} = \frac{1}{p}$, $m_T = \frac{1}{p}$ or $m_N = -p$ with no evidence of using calculus then they can gain any of or all the final 4 marks in part (a).		
	ALT	**Alternative Method for the 3rd M mark and 4th M mark**		
		$\{B(10a, 0),\ P(ap^2, 2ap) \Rightarrow\}$ $m_{BP} = \frac{2ap - 0}{ap^2 - 10a} = -p$	Finds gradient of BP and sets the result equal to the gradient of their normal	3rd M1
		$\Rightarrow p = ... \Rightarrow x = ...$ or $y = ...$	**dependent on the previous M mark** Complete method to find either the x or y coordinate of P	4th M1

		Question 9 Notes Continued
9. (b)	**Note**	Give A0 $25.4558...a^2$ without reference to $18\sqrt{2}\,a^2$
	Note	Condone one slip of either writing 9 for $10a-a$ or writing $"4\sqrt{2}"$ instead of $"4\sqrt{2}\,a"$ for the M mark in (b)
(c)	**Note**	**Way 2:** For reference, $(x-10a)^2+(-2\sqrt{2}x+20\sqrt{2}a)^2=\frac{9}{4}a^2$ $x^2-20ax+100a^2+8x^2-160ax+800a^2=\frac{9}{4}a^2$ $9x^2-180ax+900a^2=\frac{9}{4}a^2$ $9x^2-180ax+\frac{3591}{4}a^2=0$ **or** $9x^2-180ax+897.75a^2=0$ **or** $x^2-20ax+99.75a^2=0$ **or** $4x^2-80ax+399a^2=0$ $x=\frac{180a\pm\sqrt{(180a)^2-4(9)(\frac{3591}{4})a^2}}{2(9)}=\frac{180a\pm 9a}{2(9)}$ $x=\frac{189a}{18},\frac{171a}{18}=10.5a,\,9.5a$
	Note	The method $PB=\sqrt{(10a-"8a")^2+("4\sqrt{2}a")^2}$ needs to be referred to in part (c) or the result of $PB=\sqrt{(10a-"8a")^2+("4\sqrt{2}a")^2}$ needs to be used in part (c) to gain the M mark in part (c)

Question Number	Scheme	Notes	Marks
5(a)	$xy = 64 \Rightarrow y = 64x^{-1} \Rightarrow \frac{dy}{dx} = -64x^{-2}$ or $xy = 64 \Rightarrow x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ or $x = 8p, y = \frac{8}{p} \Rightarrow \frac{dy}{dx} = \frac{-8p^{-2}}{8}$	Correct $\frac{dy}{dx}$ This can be in any form, simplified or unsimplified. The parameter could be a different variable, e.g. t	B1
	$m_T = -\frac{64}{64p^2} \Rightarrow m_N = p^2$ or $m_T = -\frac{8/p}{8p} \Rightarrow m_N = p^2$ or $m_T = -p^{-2} \Rightarrow m_N = p^2$	Correct use of the perpendicular gradient rule and the point P to obtain the normal gradient	M1
		Correct normal gradient of p^2	A1
	$y - \frac{8}{p} = p^2(x - 8p)$ or $y = p^2x + c, \frac{8}{p} = p^2 \times 8p + c \Rightarrow c = ...$	Correct straight line method for normal	M1
	$p^3x - py = 8(p^4 - 1)$*	cso. (No errors, but possibly direct from the version in line 1 above)	A1*
			(5)
(b)	$p^3x - py = 8(p^4 - 1), xy = 64 \Rightarrow$ $p^3x - p\frac{64}{x} = 8(p^4 - 1)$ or $p^3\frac{64}{y} - py = 8(p^4 - 1)$	Uses both equations to obtain an equation in one variable	M1
	$p^3x^2 + 8(1 - p^4)x - 64p = 0$ or $py^2 + 8(p^4 - 1)y - 64p^3 = 0$	Correct quadratic. Must have the x^2 or y^2 term, but the x or y terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions, e.g. $p^2x^2 + \frac{8x}{p} - 8p^3x = 64$	A1
	$(x - 8p)(p^3x + 8) = 0 \Rightarrow x = ...$ or $(py - 8)(y + 8p^3) = 0 \Rightarrow y = ...$	Solves their 3TQ (usual rules) to obtain the <u>other</u> value of x or y. The <u>other</u> value must be picked out as a solution. This could be done by algebraic division... (see below)	dM1
	$x = -\frac{8}{p^3}$ $y = -8p^3$ or $\left(-\frac{8}{p^3}, -8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)
			Total 9

5(b)	Rather than solving the 3TQ for the dM1, algebraic division can be used. To score the mark the division should follow the usual rules for solution by factorisation, so in the first case, e.g. if the quadratic is correct, the quotient should be $\pm p^3 x \pm 8$, then this must lead to the other value $x_2 = \cdots$

5(b)	Note that another way to find the other value for the dM1 is to use the 'sum of roots' $= -\frac{b}{a}$, e.g. $$8p + x_2 = \frac{-8(1-p^4)}{p^3} \qquad x_2 = \cdots$$

(b) **Way 2**	$p^3x - py = 8\left(p^4 - 1\right), \left(8q, \frac{8}{q}\right) \Rightarrow$ $p^3 8q - p\frac{8}{q} = 8\left(p^4 - 1\right)$	Uses the given normal equation and the parametric form for Q to form an equation in p and q	M1
	$p^3q^2 - p = qp^4 - q$	Correct quadratic. Must have the q^2 term, but the q terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions.	A1
	$(p-q)\left(p^3q+1\right) = 0 \Rightarrow q = \ldots$	Solves their 3TQ (usual rules) to obtain the value of q. This could be done by algebraic division (condition as for main scheme)	dM1
	$q = -\frac{1}{p^3} \Rightarrow x = -\frac{8}{p^3} \quad y = -8p^3$ or $\left(-\frac{8}{p^3}, -8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)

Question Number	Scheme	Notes	Marks
4(a)	$y = 25x^{-1} \Rightarrow \frac{dy}{dx} = -25x^{-2}$ $xy = 25 \Rightarrow y + x\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = -\frac{5}{t^2}\cdot\frac{1}{5}$	Attempts a derivative expression, such as $\frac{dy}{dx} = kx^{-2}$ or $x\frac{dy}{dx} = ky$ or $\frac{dy}{dx} = \frac{\text{their}\frac{dy}{dt}}{\text{their}\frac{dx}{dt}}$	M1
	$\frac{dy}{dx} = -25x^{-2}$ or $\frac{-y}{x}$ or $\frac{-1}{t^2}$	Correct derivative.	A1
	$y - \frac{5}{t} = -\frac{1}{t^2}(x - 5t)$	Uses $y - \frac{5}{t} = (\text{their gradient})\ x\left(x - 5t\right)$ or $y = (\text{their gradient})x + c$ using $x = 5t, y = \frac{5}{t}$ in an attempt to find c. Their gradient must be a function of t for marks to be awarded.	M1
	$t^2y + x = 10t$ *	cso	A1
			(4)
Alt 4(a)	Intersect when $(10t - t^2y)y = 25$	Substitutes given line equation into the curve equation to find intersections	M1
	$\Rightarrow t^2y^2 - 10ty + 25 = 0$	Correct equation	A1
	$\Rightarrow (ty - 5)^2 = 0 \Rightarrow y = \frac{5}{t}$ is single root.	Shows the equation has a single root.	M1
	Single root when $y = \frac{5}{t}$ means the line is the tangent to H at P.	All work correct, with correct conclusion.	A1
			(4)
4(b)	$t^2(-5) + 15 = 10t$	Substitute $(15, -5)$ into equation of tangent	M1
	$(t+3)(t-1) = 0 \Rightarrow t = ..$	Solves via any valid means	M1
	$\Rightarrow t = -3, t = 1$ (or one correct point)	Both values of t **or** one correct coordinate pair.	A1
	$\left(-15, -\frac{5}{3}\right)$ and $(5,5)$	Both correct sets of coordinates.	A1
			(4)
ALT	$t_1^2y + x = 10t_1$ $t_2^2y + x = 10t_2$	Form equations with their t_1, t_2 and attempt to solve for x and y	M1
	$(x =)\frac{10t_1t_2}{(t_1 + t_2)} = 15, (y =)\frac{10}{(t_1 + t_2)} = -5$	Equates to coordinate and attempt to solve for t_1, t_2	M1
	$t_1 = -3, t_2 = 1$ (or one correct point)	Both values of t **or** one correct coordinate pair.	A1
	$\left(-15, -\frac{5}{3}\right)$ and $(5,5)$	Both correct sets of coordinates.	A1
			(4)
			Total 8

Number			
2	$f(x)=x^3-3x^2+\frac{1}{2\sqrt{x^5}}+2$		
(a)	$f(2)=...$ **and** $f(3)=...$	Attempts both f(2) and f(3)	M1
	$f(2)=-1.9116..$, $f(3)=2.032...$ **Sign change** (and $f(x)$ is continuous) therefore a **root** α exists between $x=2$ and $x=3$	Both values correct : $f(2)=-1.9116..$ (awrt -1.9), and $f(3)=2.032...$ (awrt 2.0 or e.g. $2+\frac{\sqrt{3}}{54}$) , **sign change** (or equivalent) and **conclusion**	A1
			(2)
(b)	$f'(x)=3x^2-6x-\frac{5}{4}x^{-3.5}$	M1: $x^n \to x^{n-1}$ A1: $3x^2-6x$ A1: $-\frac{5}{4}x^{-3.5}$ or equivalent un-simplified and no other terms (+ c loses this mark)	M1A1A1
	$\alpha=3-\frac{2.032075015}{8.973270821}$	Correct attempt at Newton-Raphson using their values of f(3) and f′(3).	M1
	$\alpha=2.774$	Cao (Ignore any subsequent applications)	A1
	Correct derivative followed by correct answer scores full marks in (b) **Correct answer with no working scores no marks in (b)**		
			(5)

(b)	$f'(x)=3x^2-6x-\frac{5}{4}x^{-3.5}$	M1: $x^n \to x^{n-1}$ A1: $3x^2-6x$ A1: $-\frac{5}{4}x^{-3.5}$ or equivalent un-simplified and no other terms (+ c loses this mark)	M1A1A1
	$\alpha=3-\frac{2.032075015}{8.973270821}$	Correct attempt at Newton-Raphson using their values of f(3) and f′(3).	M1
	$\alpha=2.774$	Cao (Ignore any subsequent applications)	A1
	Correct derivative followed by correct answer scores full marks in (b) **Correct answer with no working scores no marks in (b)**		
			(5)

	NB if the answer is incorrect it must be clear that both f(3) and f′(3) are being used in the Newton-Raphson process. So that just $3-\frac{f(3)}{f'(3)}$ with an incorrect answer and no other evidence scores M0.		
			Total 7

Question Number	Scheme	Notes	Marks
4.	Given $f(x) = 2x^3 - \frac{7}{x^2} + 16$, $x \neq 0$; Roots α, β: $-2 \leq \alpha \leq -1$ and $0.6 \leq \beta \leq 0.7$		
(a) **Way 1**	$f(-1.5) = ...$	Attempts to evaluate $f(-1.5)$	M1
	$f(-1.75) = ...$	**dependent on the previous M mark** Evaluates $f(-1.75)$ (and not $f(-1.25)$)	dM1
	$f(-2) = -1.75$ **or** $f(-1) = 7$ $f(-1.5) = 6.1388...$ or $\frac{221}{36}$ $f(-1.75) = 2.9955...$ or $\frac{671}{224}$ so interval is $[-2, -1.75]$	**dependent on the 2 previous marks** **Both** • $f(-2)$ correct or correct awrt (or truncated) to 1sf or $f(-1) = 7$ **and** • $f(-1.5)$ and $f(-1.75)$ correct or correct awrt (or truncated) to 1 sf **and** the correct interval stated **Note:** Allow $-2 \leq x \leq -1.75$ or $-2 < x < -1.75$ or $-2 \leq \alpha \leq -1.75$ or $-2 < \alpha < -1.75$ or $[-2, -1.75]$ or $(-2, -1.75)$ equivalent in words. Condone $-2 - -1.75$ Allow a mixture of "ends". Do not allow incorrect statements such as $-1.75 < \alpha < -2$ or $(-1.75, -2)$ or $-1.75 - -2$ unless they are recovered. Ignore the subsequent iteration of $f(-1.875)$	A1
	Note that some candidates only indicate the sign of f and not its value. **In this case the M marks can still score as defined but not the A mark.**		**(3)**
(a) **Way 2**	**Common approach in the form of a table (use the mark scheme above)** so interval is $-2 \leq \alpha \leq -1.75$ **would score full marks in part (a)**		
(b)	$f'(x) = 6x^2 + 14x^{-3}$	At least one of either $2x^3 \to \pm Ax^2$ or $-\frac{7}{x^2} \to \pm Bx^{-3}$; $A, B \neq 0$	M1
		Correct differentiation which can be simplified or un-simplified	A1
	$\left\{\beta \simeq 0.65 - \frac{f(0.65)}{f'(0.65)}\right\} \Rightarrow \beta \simeq 0.65 - \frac{-0.01879733728...}{53.51360719...}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $f(0.65)$ and $f'(0.65)$	dM1
	$\{\beta = 0.6503512623...\} \Rightarrow \beta = 0.6504$ (4 dp)	**dependent on all 3 previous marks** 0.6504 on their first iteration (Ignore any subsequent iterations)	A1 **cso** **cao**
	Correct differentiation followed by a correct answer of 0.6504 scores full marks in part (b) **Correct answer with no working scores no marks in part (b)**		**(4)**
			7

Question 4 Notes

4. (a)	**Note**	Give 2nd M0 and A0 for evaluating both $f(-1.25)$ and $f(-1.75)$
	Note	Do not allow "interval = $f(-2)$ to $f(-1.75)$" unless recovered.
	Note	A method of evaluating $f(-1.5)$ followed by $f(-1.75)$ with ***no evidence*** of evaluating at least one of either $f(-2)$ or $f(-1)$ is M1 dM1 A0.
	Note	Do not confuse the -1.75 in $f(-2) = -1.75$ with the -1.75 in $(-2, -1.75)$

		Question 4 Notes Continued
4. (b)	**dM1**	This mark can be implied by applying at least one correct ***value*** of either $f(0.65)$ or their $f'(0.65)$ (where $f'(0.65)$ is found using their $f'(x)$) to 1 significant figure in $0.65 - \dfrac{f(0.65)}{f'(0.65)}$. So just $0.65 - \dfrac{f(0.65)}{f'(0.65)}$ with an incorrect answer and no other evidence scores final dM0A0.
	Note	If you see $0.65 - \dfrac{f(0.65)}{f'(0.65)} = 0.6504$ with no algebraic differentiation, then send the response to review.
	Note	You can imply the M1 A1 marks for algebraic differentiation for either • $f'(0.65) = 6(0.65)^2 + 14(0.65)^{-3}$ • $f'(0.65)$ applied correctly in $\beta \simeq 0.65 - \dfrac{2(0.65)^3 - \dfrac{7}{(0.65)^2} + 16}{6(0.65)^2 + 14(0.65)^{-3}}$
	Note	**Differentiating INCORRECTLY to give** $f'(x) = 6x^2 - 14x^{-3}$ leads to $\beta \simeq 0.65 - \dfrac{-0.01879733728...}{-48.44360719...} = 0.6496119749... = 0.6496$ (4 dp) **This response should be awarded M1 A0 dM1 A0**
	Note	**Differentiating INCORRECTLY to give** $6x^2 - 14x^{-3}$ and $\beta \simeq 0.65 - \dfrac{f(0.65)}{f'(0.65)} = 0.6496$ is **M1 A0 dM1 A0**

Question Number	Scheme	Notes	Marks
2(a)	$f(x)=7\sqrt{x}-\frac{1}{2}x^3-\frac{5}{3x}\qquad x>0$		
	$f(2.8)=0.1420022...$ $f(2.9)=-0.8486421...$	Attempts both f(2.8) and f(2.9) with at least one correct to 1 s.f.	M1
	Sign change (positive, negative) and $f(x)$ is **continuous** therefore (a **root**) α is between $x=2.8$ and $x=2.9$	Both f(2.8) = awrt 0.1 (or truncated) and f(2.9) = awrt –0.8 (or truncated), sign change, continuous and minimal conclusion.	A1
			(2)
(b)(i)	$f'(x)=\frac{7}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^2+\frac{5}{3x^2}$	$x^n\rightarrow x^{n-1}$ at least once	M1
		Correct derivative	A1
(b)(ii)	$x_1=2.8-\frac{f(2.8)}{f'(2.8)}=2.8-\frac{0.142002276...}{-9.4557649...}$	Correct application of Newton-Raphson. If no substitution/values see accept a correct statement followed by a value for the attempt.	M1
	$=2.815$	cao following a correct derivative.	A1
			(4)
(c)	$\frac{2.9-\alpha}{0.8486421875...}=\frac{\alpha-2.8}{0.1420022762...}$	Any correct or implied linear interpolation statement.	B1
	$\alpha=\frac{2.8\times0.8486421875...+2.9\times0.1420022762...}{0.8486421875...+0.1420022762...}=...$ Rearranges an equation suitable form (e.g. allow sign errors in interpolation statement) to give $\alpha=\ldots$		M1
	$=2.814$	cao	A1
			(3)
Alt (c)	$\frac{x}{0.1420022762...}=\frac{0.1}{0.1420022762...+0.8486421875...}$	Any correct or implied linear interpolation statement for x distance.	B1
	$\alpha=2.8+x=2.8+\frac{0.4\times0.1420022762...}{0.8486421875...+0.1420022762...}=...$ Rearranges and adds 2.8 to give $\alpha=\ldots$		M1
	$=2.814$	cao	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
8.	$H: xy = 16;\ P\left(4t, \frac{4}{t}\right), t \neq 0,$ and $A: t = 2$ lies on H. $A(8, 2)$		
(a)	$y = \frac{16}{x} = 16x^{-1} \Rightarrow \frac{dy}{dx} = -16x^{-2}$ or $-\frac{16}{x^2}$	$\frac{dy}{dx} = \pm kx^{-2}; k \neq 0$	M1
	$xy = 16 \Rightarrow x\frac{dy}{dx} + y = 0$	Uses implicit differentiation to give $\pm x\frac{dy}{dx} \pm y$	
	$x = 4t,\ y = \frac{4}{t} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\left(\frac{4}{t^2}\right)\left(\frac{1}{4}\right)$	their $\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dy}{dt}}$; **Condone** $p \equiv t$	
	So at P, $m_T = -\frac{1}{t^2}$	Correct calculus work leading to $m_T = -\frac{1}{t^2}$	A1
	So, $m_N = t^2$	Applies $m_N = \frac{-1}{m_T}$, where m_T is found using calculus	M1
	• $y - \frac{4}{t} = "t^2"(x - 4t)$ • $\frac{4}{t} = "t^2"(4t) + c \Rightarrow y = "t^2"x +$ their c	Correct straight line method for an equation of a normal where $m_N\ (\neq m_T)$ is found by using calculus	M1
	Correct algebra leading to $ty - t^3x = 4 - 4t^4$ *	Correct solution only	A1 cso
			(5)
(b)	$\{t = 2 \Rightarrow\}$ **N**: $2y - 8x = 4 - 64$ $\{\Rightarrow y = 4x - 30\}$	Uses $t = 2$ to find the equation of the normal to H at A	M1
	• $x(4x - 30) = 16$ $\{\Rightarrow 2x^2 - 15x - 8 = 0\}$ • $\left(\frac{y + 30}{4}\right)y = 16$ $\{\Rightarrow y^2 + 30y - 64 = 0\}$ • $\frac{4}{t} = 4(4t) - 30$ $\{\Rightarrow 8t^2 - 15t - 2 = 0\}$	Substitutes the equation of the normal into the equation of the curve H to obtain an equation in x only or y only or t only	M1
	• $(x - 8)(2x + 1) = 0 \Rightarrow x_B = -\frac{1}{2}$ • $(y - 2)(y + 32) = 0 \Rightarrow y_B = -32$ • $(t - 2)(8t + 1) = 0 \Rightarrow t_B = -\frac{1}{8}$	**dependent on the first two M marks** Solves their 3 TQ $= 0$ to obtain a value for the x (or y) coordinate of B or a value of t at B	ddM1
	$B(-0.5, -32)$	Correct coordinates for B	A1
	$AB = \sqrt{(8 - -0.5)^2 + (2 - -32)^2}$	**dependent on the second M mark** Correct Pythagoras method to find the length of AB	dM1
	$= \frac{17\sqrt{17}}{2}$ or $\frac{\sqrt{4913}}{2}$ or $\sqrt{\frac{4913}{4}}$ or $\sqrt{1228.25}$	Correct exact length	A1
			(6)
(c)	$y - 2 = -\frac{1}{4}(x - 8)$ and $x = 0 \Rightarrow y_C = 2 + 2 = 4$	Finds the equation of the tangent at $(8, 2)$ to H, and sets $x = 0$ to find $y_C = ...$	M1
	$AC = \sqrt{(8 - 0)^2 + (2 - 4)^2}$ $\{= \sqrt{68}\}$ Area $ABC = \frac{1}{2}\left(\frac{17\sqrt{17}}{2}\right)\left(\sqrt{68}\right)$	Uses the points $(8, 2)$, $(-0.5, -32)$ and $(0, 4)$ in a complete method to find the area of triangle ABC	M1
	$= 144.5$ or $\frac{289}{2}$	Correct answer	A1
			(3)
			14

		Question 8 Notes
8. (b)	**Note**	The correct coordinates of B can be implied. e.g. embedded in the distance expression for AB
	Note	An incorrect **N**: $y = 4x + 30$ leads to the correct length AB for $A(-8, -2)$ and $B(0.5, 32)$
	Note	Condone final dM1 for $x_B = -\frac{1}{2}$ leading to $B(-2, -8)$ and $AB = \sqrt{(8--2)^2 + (2--8)^2}$
(c)	**Note**	Give 1st M0 for setting $x = 0$ in the equation of the normal to find $y_C = ...$
	Note	The 2nd M mark can only be gained by using all 3 correct points $(8, 2)$, $(-0.5, -32)$ and $(0, 4)$. Complete area methods include • Area $ABC = \frac{1}{2}\left(\frac{17\sqrt{17}}{2}\right)\left(\sqrt{68}\right)\{= 144.5\}$ • AB crosses y-axis at $(0, -30)$ and so Area $ABC = \frac{1}{2}(34)\left(\frac{1}{2}\right) + \frac{1}{2}(34)(8)\{= 8.5 + 136 = 144.5\}$ • Area $ABC = \frac{1}{2}\begin{vmatrix} 8 & -0.5 & 0 & 8 \\ 2 & -32 & 4 & 2 \end{vmatrix} = \frac{1}{2}\lvert(-256 - 2 + 0) - (-1 + 0 + 32)\rvert \left\{= \frac{1}{2}\lvert(-289)\rvert = 144.5\right\}$ • Area $ABC = (32 + 4)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(32 + 2)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(32 + 4)\left(\frac{1}{2}\right) - \frac{1}{2}(2)(8)$ $\{= 306 - 144.5 - 9 - 8 = 144.5\}$ • Area $ABC = \frac{1}{2}(8 + 8.5)(36) - \frac{1}{2}(32 + 2)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(2)(8)$ $\{= 297 - 144.5 - 8 = 144.5\}$
	Note	**Helpful Sketch** 