9. (i)
$$f(n) = 7^n(3n + 1) - 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is a multiple of 9

(6)

(ii) A sequence of numbers is defined by

$$u_1 = 2 \qquad u_2 = 6$$

$$u_{n+2} = 3u_{n+1} - 2u_n \qquad n \in \mathbb{Z}^+$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 2(2^n - 1)$$

(6)

3. Prove by induction that for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2}{r(r + 1)(r + 2)} = \frac{1}{2} - \frac{1}{(n + 1)(n + 2)}$$

(5)

9. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \left(4r^3 - 3r^2 + r\right) = n^3(n + 1)$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 5^{2n} + 3n - 1$$

is divisible by 9

(6)

8. Prove by induction that $4^{n+2} + 5^{2n+1}$ is divisible by 21 for all positive integers n.

(6)

9. (i) A sequence of numbers is defined by

$$u_1 = 3$$

$$u_{n+1} = 2u_n - 2^{n+1} \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{N}$

$$u_n = 5 \times 2^{n-1} - n \times 2^n$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{N}$

$$f(n) = 5^{n+2} - 4n - 9$$

is divisible by 16

(5)