Leave blank

8. Prove by induction that $4^{n+2} + 5^{2n+1}$ is divisible by 21 for all positive integers n.

(6)

Leave blank

10. (i) A sequence of positive numbers is defined by

$$u_1 = 5$$
$$u_{n+1} = 3u_n + 2, \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 2 \times (3)^n - 1$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3 + 2n)}{3^n}$$

(6)

blank

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2r^2 - 1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2}$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 12^n + 2 \times 5^{n-1}$$

is divisible by 7

(6)

4.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

(a) Describe the single geometrical transformation represented by the matrix **A**.

(2)

The matrix **B** represents a rotation of 210° anticlockwise about centre (0, 0).

(b) Write down the matrix **B**, giving each element in exact form.

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Find **C**.

(2)

The hexagon H is transformed onto the hexagon H' by the matrix **C**.

(d) Given that the area of hexagon H is 5 square units, determine the area of hexagon H'

(2)

2:

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The matrix **P** represents the transformation P.

(a) Describe P fully as a single geometrical transformation.

(2)

$$\mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

The matrix **Q** represents the transformation Q.

(b) Describe Q fully as a single geometrical transformation.

(2)

The transformation R is a reflection in the line $y = -x$

(c) Write down the matrix representing R.

(1)

The transformation R maps the point $(4, 3)$ to the point A.

(d) Determine the coordinates of A.

(1)

The transformation P followed by the transformation Q maps the point B to the point A.

(e) Determine the coordinates of B.

(2)

Leave blank

1.

$$\mathbf{A} = \begin{pmatrix} 3 & a \\ -2 & -2 \end{pmatrix}$$

where a is a non-zero constant and $a \neq 3$

(a) Determine $\mathbf{A}^{-1}$ giving your answer in terms of a.

(2)

Given that $\mathbf{A} + \mathbf{A}^{-1} = \mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a.

(3)

Leave blank

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^n = \begin{pmatrix} 1+4n & -8n \\ 2n & 1-4n \end{pmatrix}$$

(5)

(ii) A sequence of positive numbers is defined by

$$u_1 = 8, \quad u_2 = 40$$
$$u_{n+2} = 8u_{n+1} - 12u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 6^n + 2^n$$

(5)

Leave blank

3.

$$\mathbf{A} = \begin{pmatrix} 4 & -2 \\ a & -3 \end{pmatrix}$$

where a is a real constant and $a \neq 6$

(a) Find $\mathbf{A}^{-1}$ in terms of a.

(3)

Given that $\mathbf{A} + 2\mathbf{A}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(b) find the value of a.

(3)

3. $\mathbf{M} = \begin{pmatrix} k & k \\ 3 & 5 \end{pmatrix}$ where k is a non-zero constant

(a) Determine $\mathbf{M}^{-1}$, giving your answer in simplest form in terms of k.

(2)

Hence, given that $\mathbf{N}^{-1} = \begin{pmatrix} k & k \\ 4 & -1 \end{pmatrix}$

(b) determine $(\mathbf{MN})^{-1}$, giving your answer in simplest form in terms of k.

(2)

6.

(i) $\mathbf{B} = \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}$, $\mathbf{Y} = \begin{pmatrix} 4 & -2 \\ 1 & 0 \end{pmatrix}$

(a) Find $\mathbf{B}^{-1}$.

(2)

The transformation represented by **Y** is equivalent to the transformation represented by **B** followed by the transformation represented by the matrix **A**.

(b) Find **A**.

(2)

(ii) $\mathbf{M} = \begin{pmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{pmatrix}$

The matrix **M** represents an enlargement scale factor k, centre (0, 0), where $k > 0$, followed by a rotation anti-clockwise through an angle θ about (0, 0).

(a) Find the value of k.

(2)

(b) Find the value of θ.

(2)

6. (a) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & (2^n - 1)r \\ 0 & 2^n \end{pmatrix}$$

where r is a constant.

(4)

$$\mathbf{M} = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} \qquad \mathbf{N} = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^4$$

The transformation represented by matrix **M** followed by the transformation represented by matrix **N** is represented by the matrix **B**

(b) (i) Determine **N** in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c and d are integers.

(ii) Determine **B**

(3)

Hexagon S is transformed onto hexagon S' by matrix **B**

(c) Given that the area of S' is 720 square units, determine the area of S

(2)