blank

7. The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}$$

The transformation represented by **A** maps triangle T onto triangle T'

Given that the area of triangle T is 23 cm^2

(a) determine the area of triangle T'

(2)

The point P has coordinates $(3p + 2, 2p - 1)$ where p is a constant. The transformation represented by **A** maps P onto the point P' with coordinates $(17, -18)$

(b) Determine the value of p.

(2)

Given that

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(c) describe fully the single geometrical transformation represented by matrix **B**

(2)

The transformation represented by matrix **A** followed by the transformation represented by matrix **C** is equivalent to the transformation represented by matrix **B**

(d) Determine **C**

(3)

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6. The rectangular hyperbola, H, has cartesian equation

$$xy = 36$$

The three points $P\left(6p, \frac{6}{p}\right)$, $Q\left(6q, \frac{6}{q}\right)$ and $R\left(6r, \frac{6}{r}\right)$, where p, q and r are distinct, non-zero values, lie on the hyperbola H.

(a) Show that an equation of the line PQ is

$$pqy + x = 6(p + q)$$

(4)

Given that PR is perpendicular to QR,

(b) show that the normal to the curve H at the point R is parallel to the line PQ.

(6)

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8. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ lies on C.

(a) Using calculus, show that the normal to C at P has equation

$$y + tx = at^3 + 2at$$

(5)

The point S is the focus of the parabola C.

The point B lies on the positive x-axis and $OB = 5OS$, where O is the origin.

(b) Write down, in terms of a, the coordinates of the point B.

(1)

A circle has centre B and touches the parabola C at two distinct points Q and R.

Given that $t \neq 0$,

(c) find the coordinates of the points Q and R.

(4)

(d) Hence find, in terms of a, the area of triangle BQR.

(2)

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1. Given that

$$2z^3 - 5z^2 + 7z - 6 \equiv (2z - 3)(z^2 + az + b)$$

where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Given that z is a complex number, find the three exact roots of the equation

$$2z^3 - 5z^2 + 7z - 6 = 0$$

(3)

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4. $z_1 = p + 5\mathrm{i}, \quad z_2 = 9 + 8\mathrm{i} \quad \text{and} \quad z_3 = \dfrac{z_1}{z_2}$

where p is a real constant.

(a) Determine z_3 in the form $x + \mathrm{i}y$, where x and y are in terms of p

(3)

(b) Determine the exact value of the modulus of z_2

(1)

Given that the argument of z_1 is $\dfrac{\pi}{3}$

(c) (i) determine the exact value of p

(ii) determine the exact value of the modulus of z_3

(3)

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5.

$$\mathbf{A} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation represented by the matrix **A**.

(3)

(b) Hence write down the matrix $\mathbf{A}^6$

(1)

The transformation represented by the matrix **C** followed by the transformation represented by the matrix **B** is equivalent to the transformation represented by the matrix **A**.

Given that

$$\mathbf{B} = \begin{pmatrix} 2\sqrt{3} & -7 \\ -4 & 5\sqrt{3} \end{pmatrix}$$

(c) find the matrix **C**, giving your answer in simplest form.

(4)

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4. $$f(z) = 6z^3 + Az^2 + Bz + C$$

where A, B and C are integers.

Given that $z = \frac{2}{3} + \frac{\sqrt{17}}{3}i$ is a root of the equation $f(z) = 0$

(a) write down the other complex root of the equation $f(z) = 0$

(1)

Given that $z = -\frac{3}{2}$ is also a root of the equation $f(z) = 0$

(b) determine the value of A, the value of B and the value of C.

(5)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

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8. A parabola C has equation $y^2 = 4ax$ where a is a positive constant.

The point S is the focus of C

The line l_1 with equation $y = k$ where k is a positive constant, intersects C at the point P

(a) Show that

$$PS = \frac{k^2 + 4a^2}{4a}$$

(3)

The line l_2 passes through P and intersects the directrix of C on the x-axis.

The line l_2 intersects the y-axis at the point A

(b) Show that the y coordinate of A is $\dfrac{4a^2k}{k^2 + 4a^2}$

(3)

The line l_1 intersects the directrix of C at the point B

Given that the areas of triangles BPA and OSP, where O is the origin, satisfy the ratio

$$\text{area } BPA : \text{area } OSP = 4k^2 : 1$$

(c) determine the exact value of a

(5)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The rectangular hyperbola H has equation $xy = 20$

The point $P\left(2t\sqrt{a}, \dfrac{2\sqrt{a}}{t}\right)$, $t \neq 0$, where a is a constant, is a general point on H

(a) State the value of a

(1)

(b) Show that the normal to H at the point P has equation

$$ty - t^3x - 2\sqrt{5}\left(1 - t^4\right) = 0$$

(4)

The points A and B lie on H

The point A has parameter $t = c$ and the point B has parameter $t = -\dfrac{1}{2c}$, where c is a constant.

The normal to H at A meets H again at B

(c) Determine the possible values of c

(4)

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2. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & 4 \\ -k & 2k \\ 3 & 0 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) find the matrix **AB**,

(2)

(b) find the exact value of k for which $\det(\mathbf{AB}) = 0$

(2)

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5.

$$f(x) = (9x^2 + d)\left(x^2 - 8x + (10d + 1)\right)$$

where d is a positive constant.

(a) Find the four roots of $f(x)$ giving your answers in terms of d.

(3)

Given $d = 4$

(b) Express these four roots in the form $a + ib$, where $a, b \in \mathbb{R}$.

(2)

(c) Show these four roots on a single Argand diagram.

(2)

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