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4. The rectangular hyperbola H has parametric equations

$$x = 4t, \quad y = \frac{4}{t}$$

The straight line with equation $3y - 2x = 10$ intersects H at the points A and B.

Given that the point A is above the x-axis,

(a) find the coordinates of the point A and the coordinates of the point B.

(5)

(b) Find the coordinates of the midpoint of AB.

(2)

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7.

$$f(x) = x^{\frac{3}{2}} + x - 3$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval $[1, 2]$

(2)

(b) Starting with the interval $[1, 2]$, use interval bisection twice to show that α lies in the interval $[1.25, 1.5]$

(3)

(c) (i) Determine $f'(x)$

(ii) Using 1.375 as a first approximation for α, apply the Newton-Raphson process once to $f(x)$ to determine a second approximation for α, giving your answer to 3 decimal places.

(3)

(d) Use linear interpolation once on the interval $[1.25, 1.5]$ to obtain a different approximation for α, giving your answer to 3 decimal places.

(3)

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7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The rectangular hyperbola H has equation $xy = 36$

The point $P(4, 9)$ lies on H

(a) Show, using calculus, that the normal to H at P has equation

$$4x - 9y + 65 = 0$$

(4)

The normal to H at P crosses H again at the point Q

(b) Determine an equation for the tangent to H at Q, giving your answer in the form $y = mx + c$ where m and c are rational constants.

(5)

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10. The rectangular hyperbola H has equation $xy = 144$. The point P, on H, has coordinates $\left(12p, \frac{12}{p}\right)$, where p is a non-zero constant.

(a) Show, by using calculus, that the normal to H at the point P has equation

$$y = p^2x + \frac{12}{p} - 12p^3$$

(5)

Given that the normal through P crosses the positive x-axis at the point Q and the negative y-axis at the point R,

(b) find the coordinates of Q and the coordinates of R, giving your answers in terms of p.

(3)

(c) Given also that the area of triangle OQR is 512, find the possible values of p.

(5)

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9. The parabola C has cartesian equation $y^2 = 4ax$, where a is a positive constant.

The point $P(ap^2, 2ap)$ lies on C.

The line l is the normal to C at the point P.

The line l passes through the point B with coordinates $(10a, 0)$.

Given that $p > 0$

(a) use calculus to find, in terms of a only, the coordinates of P.

(7)

The point S is the focus of the parabola C.

(b) Find, in terms of a, the exact area of triangle SBP.

(2)

A circle has equation

$$(x - 10a)^2 + y^2 = \frac{9}{4}a^2$$

Given that the line l cuts this circle at the point R, where $y > 0$

(c) find, in terms of a, the distance PR.

(3)

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5. The rectangular hyperbola H has equation $xy = 64$

The point $P\left(8p, \frac{8}{p}\right)$, where $p \neq 0$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$p^3x - py = 8(p^4 - 1)$$

(5)

The normal to H at P meets H again at the point Q.

(b) Determine, in terms of p, the coordinates of Q, giving your answers in simplest form.

(4)

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4. A rectangular hyperbola H has equation $xy = 25$

The point $P\left(5t, \frac{5}{t}\right)$, $t \neq 0$, is a general point on H.

(a) Show that the equation of the tangent to H at P is $t^2y + x = 10t$

(4)

The distinct points Q and R lie on H. The tangent to H at the point Q and the tangent to H at the point R meet at the point $(15, -5)$.

(b) Find the coordinates of the points Q and R.

(4)

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2. $$f(x) = x^3 - 3x^2 + \frac{1}{2\sqrt{x^5}} + 2, \quad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

(2)

(b) Taking 3 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to find a second approximation to α. Give your answer to 3 decimal places.

(5)

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4.

$$f(x) = 2x^3 - \frac{7}{x^2} + 16, \quad x \neq 0$$

The equation $f(x) = 0$ has a single root α between $x = -2$ and $x = -1$

(a) Starting with the interval $[-2, -1]$, use interval bisection twice to find an interval of width 0.25 that contains α.

(3)

The equation $f(x) = 0$ also has a single root β in the interval $[0.6, 0.7]$.

(b) Taking 0.65 as a first approximation to β, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to β, giving your answer to 4 decimal places.

(4)

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2.

$$f(x) = 7\sqrt{x} - \frac{1}{2}x^3 - \frac{5}{3x} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval $[2.8, 2.9]$

(2)

(b) (i) Find $f'(x)$.

(ii) Hence, using $x_0 = 2.8$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(4)

(c) Use linear interpolation once on the interval $[2.8, 2.9]$ to find another approximation to α. Give your answer to 3 decimal places.

(3)

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8. A rectangular hyperbola, H, has Cartesian equation $xy = 16$

The point $P\left(4t, \frac{4}{t}\right)$, $t \neq 0$, lies on H.

(a) Use calculus to show that an equation of the normal to H at P is

$$ty - t^3x = 4 - 4t^4$$

(5)

The point A on H has parameter $t = 2$

The normal to H at A meets H again at the point B.

(b) Determine the exact value of the length of AB.

(6)

The tangent to H at A meets the y-axis at the point C.

(c) Determine the exact area of triangle ABC.

(3)

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