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6. The quadratic equation

$$Ax^2 + 5x - 12 = 0$$

where A is a constant, has roots α and β

(a) Write down an expression in terms of A for

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(2)

The equation

$$4x^2 - 5x + B = 0$$

where B is a constant, has roots $\alpha - \frac{3}{\beta}$ and $\beta - \frac{3}{\alpha}$

(b) Determine the value of A

(3)

(c) Determine the value of B

(3)

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5. In the interval $2 < x < 3$, the equation

$$6 - x^2 \cos\left(\frac{x}{5}\right) = 0, \text{ where } x \text{ is measured in radians}$$

has exactly one root α.

(a) Starting with the interval [2, 3], use interval bisection twice to find an interval of width 0.25 which contains α.

(4)

(b) Use linear interpolation once on the interval [2, 3] to find an approximation to α. Give your answer to 2 decimal places.

(3)

8. $$f(x) = 2x^{-\frac{2}{3}} + \frac{1}{2}x - \frac{1}{3x-5} - \frac{5}{2} \qquad x \neq \frac{5}{3}$$

The table below shows values of f(x) for some values of x, with values of f(x) given to 4 decimal places where appropriate.

x	1	2	3	4	5
f(x)	0.5		–0.2885		0.5834

(a) Complete the table giving the values to 4 decimal places.

(2)

The equation f(x) = 0 has exactly one positive root, α.

Using the values in the completed table and explaining your reasoning,

(b) determine an interval of width one that contains α.

(2)

(c) Hence use interval bisection twice to obtain an interval of width 0.25 that contains α.

(3)

Given also that the equation f(x) = 0 has a negative root, β, in the interval [–1, –0.5]

(d) use linear interpolation once on this interval to find an approximation for β.

Give your answer to 3 significant figures.

(3)

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7. The equation $3x^2 + px - 5 = 0$, where p is a constant, has roots α and β.

(a) Determine the value of

(i) $\alpha\beta$

(ii) $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

(3)

(b) Obtain an expression, in terms of p, for

(i) $\alpha + \beta$

(ii) $\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right)$

(3)

Given that

$$\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = 2\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

(c) determine the value of p.

(1)

(d) Using the value of p found in part (c), obtain a quadratic equation, with integer coefficients, that has roots $\left(\alpha + \frac{1}{\beta}\right)$ and $\left(\beta + \frac{1}{\alpha}\right)$

(2)

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1. $f(x) = 2^x - 10 \sin x - 2$, where x is measured in radians

(a) Show that $f(x) = 0$ has a root, α, between 2 and 3

(2)

(b) Use linear interpolation once on the interval [2, 3] to find an approximation to α. Give your answer to 3 decimal places.

(3)

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6. The quadratic equation

$$2x^2 + x + 4 = 0$$

has roots α and β

Without solving the quadratic equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) find the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(4)

(c) find a quadratic equation that has roots

$$\left(\alpha^3 + \frac{1}{\beta}\right) \text{ and } \left(\beta^3 + \frac{1}{\alpha}\right)$$

giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers.

(4)

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5. The quadratic equation

$$2x^2 - 3x + 7 = 0$$

has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine the value of $\alpha^2 + \beta^2$

(2)

(c) find a quadratic equation which has roots

$$\left(\alpha - \frac{1}{\beta^2}\right) \text{ and } \left(\beta - \frac{1}{\alpha^2}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined.

(6)

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3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(z) = 4z^3 + pz^2 - 24z + 108$$

where p is a constant.

Given that -3 is a root of the equation $f(z) = 0$

(a) determine the value of p

(2)

(b) using algebra, solve $f(z) = 0$ completely, giving the roots in simplest form,

(4)

(c) determine the modulus of the complex roots of $f(z) = 0$

(2)

(d) show the roots of $f(z) = 0$ on a single Argand diagram.

(2)

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1. The quadratic equation

$$3x^2 - 5x + 1 = 0$$

has roots α and β.

Without solving the quadratic equation, find the exact value of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

(4)

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3.

$$f(x) = x^3 - 5\sqrt{x} - 4x + 7 \qquad x \geqslant 0$$

The equation $f(x) = 0$ has a root α in the interval $[0.25, 1]$

(a) Use linear interpolation once on the interval $[0.25, 1]$ to determine an approximation to α, giving your answer to 3 decimal places.

(3)

The equation $f(x) = 0$ has another root β in the interval $[1.5, 2.5]$

(b) Determine $f'(x)$

(2)

(c) Hence, using $x_0 = 1.75$ as a first approximation to β, apply the Newton–Raphson process once to $f(x)$ to determine a second approximation to β, giving your answer to 3 decimal places.

(2)

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6.

$$f(x) = \frac{2(x^3 + 3)}{\sqrt{x}} - 9, \quad x > 0$$

The equation $f(x) = 0$ has two real roots α and β, where $0.4 < \alpha < 0.5$ and $1.2 < \beta < 1.3$

(a) Taking 0.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(5)

(b) Use linear interpolation once on the interval [1.2, 1.3] to find an approximation to β, giving your answer to 3 decimal places.

(4)

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