8 (a)	$x = e^u \quad \frac{dx}{du} = e^u$ or $\frac{du}{dx} = e^{-u}$ or $\frac{dx}{du} = x$	B1
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{-u}\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = -e^{-u}\frac{du}{dx}\frac{dy}{du} + e^{-u}\frac{d^2y}{du^2}\frac{du}{dx} = e^{-2u}\left(-\frac{dy}{du} + \frac{d^2y}{du^2}\right)$	M1A1
	$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 8y = 4\ln x$	
	$e^{2u} \times e^{-2u}\left(-\frac{dy}{du} + \frac{d^2y}{du^2}\right) + 3e^u \times e^{-u}\frac{dy}{du} - 8y = 4\ln\left(e^u\right)$	dM1
	$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u$ *	A1*cso (6)
(b)	$m^2 + 2m - 8 = 0$	
	$(m+4)(m-2) = 0, \quad m = -4, 2$	M1A1
	CF $= Ae^{-4u} + Be^{2u}$	A1
	PI: try $y = au + b$ (or $y = cu^2 + au + b$ different derivatives, c = 0)	
	$\frac{dy}{du} = a \quad \frac{d^2y}{du^2} = 0$	M1
	$0 + 2a - 8(au + b) = 4u$	
	$a = -\frac{1}{2} \quad b = -\frac{1}{8}$	dM1A1
	$\therefore y = Ae^{-4u} + Be^{2u} - \frac{1}{2}u - \frac{1}{8}$	B1ft (7)
(c)	$y = Ax^{-4} + Bx^2 - \frac{1}{2}\ln x - \frac{1}{8}$	B1 (1)
		[14]

Question Number	Scheme	Notes	Marks
2 **Way 1**	$w=\dfrac{1}{z+1}\Rightarrow z=\dfrac{1-w}{w}$	Makes z the subject and obtains $z=\dfrac{\pm1\pm w}{w}$	M1
	$z=\dfrac{1-(u+\mathrm{i}v)}{u+\mathrm{i}v}\times\dfrac{u-\mathrm{i}v}{u-\mathrm{i}v}$	Replaces w with $u+\mathrm{i}v$ and multiplies top and bottom by complex conjugate of their denominator. This statement is sufficient.	M1
	$x=0\Rightarrow\dfrac{u-(u^2+v^2)}{u^2+v^2}=0$	Equates real part to zero	M1
	$\Rightarrow u^2+v^2-u=0$	Correct equation connecting u and v	A1 **M1 on ePEN**
	Centre $\left(\dfrac{1}{2},0\right)$ **or** radius $\dfrac{1}{2}$	One correct **but must follow the use of a correct circle equation**	A1cso
	Centre $\left(\dfrac{1}{2},0\right)$ **and** radius $\dfrac{1}{2}$	Both correct **but must follow the use of a correct circle equation**	A1cso
			(6)

8(a)	E.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}t}\dfrac{\mathrm{d}t}{\mathrm{d}x}=\dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}t}$ or $t=\ln x\Rightarrow x=\mathrm{e}^t\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^t\dfrac{\mathrm{d}t}{\mathrm{d}y}$ or $t=\ln x\Rightarrow\dfrac{\mathrm{d}t}{\mathrm{d}y}=\dfrac{1}{x}\dfrac{\mathrm{d}x}{\mathrm{d}y}$ etc.	B1
	E.g. $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}t}+\dfrac{1}{x}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}\dfrac{\mathrm{d}t}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^t\dfrac{\mathrm{d}t}{\mathrm{d}y}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-t}\dfrac{\mathrm{d}y}{\mathrm{d}t}\Rightarrow\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^{-t}\dfrac{\mathrm{d}t}{\mathrm{d}x}\dfrac{\mathrm{d}y}{\mathrm{d}t}+\mathrm{e}^{-t}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}\dfrac{\mathrm{d}t}{\mathrm{d}x}$ or $x\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}t}\Rightarrow x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}\dfrac{\mathrm{d}t}{\mathrm{d}x}$ etc	M1
	E.g. $\left(t=\ln x\Rightarrow\dfrac{\mathrm{d}t}{\mathrm{d}x}=\dfrac{1}{x}\Rightarrow\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}t}+\dfrac{1}{x}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}\dfrac{\mathrm{d}t}{\mathrm{d}x}=-\mathrm{e}^{-2t}\dfrac{\mathrm{d}y}{\mathrm{d}t}+\mathrm{e}^{-2t}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$ or $\left(x=\mathrm{e}^t\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{e}^t\Rightarrow\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^{-t}\dfrac{\mathrm{d}t}{\mathrm{d}x}\dfrac{\mathrm{d}y}{\mathrm{d}t}+\mathrm{e}^{-t}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}\dfrac{\mathrm{d}t}{\mathrm{d}x}=-\mathrm{e}^{-2t}\dfrac{\mathrm{d}y}{\mathrm{d}t}+\mathrm{e}^{-2t}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$ or $x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}\dfrac{\mathrm{d}t}{\mathrm{d}x}\Rightarrow\mathrm{e}^t\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\dfrac{1}{\mathrm{e}^t}\dfrac{\mathrm{d}y}{\mathrm{d}t}=\dfrac{1}{\mathrm{e}^t}\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$ $\Rightarrow\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\mathrm{e}^{-2t}\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}-\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)$*	A1*
		(3)

(b)	$x^2\dfrac{d^2y}{dx^2}-2y=1+4\ln x-2(\ln x)^2 \Rightarrow e^{2t}\times e^{-2t}\left(\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}\right)-2y=1+4t-2t^2$ $\Rightarrow \dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}-2y=1+4t-2t^2$ *	B1*
		(1)
(c)	$m^2-m-2=0\Rightarrow m=2,-1$ $y=Ae^{2t}+Be^{-t}$	M1 A1
	$y=at^2+bt+c\Rightarrow\dfrac{dy}{dt}=2at+b\Rightarrow\dfrac{d^2y}{dt^2}=2a$	M1
	$\Rightarrow 2a-2at-b-2at^2-2bt-2c=1+4t-2t^2$ $2a=2\Rightarrow a=1$ $-2a-2b=4\Rightarrow b=-3$ $2a-b-2c=1\Rightarrow c=2$	dM1
	$y=Ae^{2t}+Be^{-t}+t^2-3t+2$	A1
		(5)
(d)	$y=Ax^2+Bx^{-1}+(\ln x)^2-3\ln x+2$	B1ft
		(1)
		Total 10

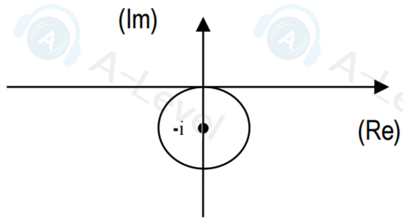
Question Number	Scheme	Notes	Marks
9(a)	$\cos 5\theta+i\sin 5\theta=(\cos\theta+i\sin\theta)^5$ $=\cos^5\theta+{}^5C_1\cos^4\theta(i\sin\theta)+{}^5C_2\cos^3\theta(i\sin\theta)^2+{}^5C_3\cos^2\theta(i\sin\theta)^3+{}^5C_4\cos\theta(i\sin\theta)^4+(i\sin\theta)^5$		M1
	$(\sin 5\theta=)5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$	M1: Equates imaginary parts A1: Correct expression	M1 A1
	$=5\sin\theta(1-\sin^2\theta)^2-10\sin^3\theta(1-\sin^2\theta)+\sin^5\theta$ $=5\sin\theta-10\sin^3\theta+5\sin^5\theta-10\sin^3\theta+10\sin^5\theta+\sin^5\theta$ $=16\sin^5\theta-20\sin^3\theta+5\sin\theta$	M1: Uses $\cos^2\theta=1-\sin^2\theta$ and reaches an expression of the correct form A1: Correct expression or $a=16,\ \ b=-20,\ \ c=5$ $\sin 5\theta$ must have been seen at some point	M1 A1
			(5)
(b)	$\left(\text{For } \theta=\dfrac{k\pi}{5},\ k\in\mathbb{Z},\ \sin 5\theta=\sin k\pi=0\Rightarrow\right)$ $16\sin^5\theta-20\sin^3\theta+5\sin\theta=0$	Sets their answer to (a) = 0 Could be implied	M1
	$\sin\theta(16\sin^4\theta-20\sin^2\theta+5)=0$ $\Rightarrow\sin^2\theta=\dfrac{-(-20)\pm\sqrt{(-20)^2-4(16)(5)}}{2(16)}=\dfrac{20\pm\sqrt{80}}{32}$	Solves their quadratic in $\sin^2\theta$ to find both roots (usual rules)	M1
	$\Rightarrow\sin^2\dfrac{k\pi}{5}=\dfrac{5\pm\sqrt{5}}{8},\ 0$	A1: $\dfrac{5\pm\sqrt{5}}{8}$ oe B1: 0	A1, B1
			(4)
			Total 9

6	$x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+3y=x^2$		
(a)	$x=e^t \Rightarrow \frac{dx}{dy}=e^t\frac{dt}{dy} \Rightarrow \frac{dy}{dx}=e^{-t}\frac{dy}{dt}$	M1: Attempt first derivative using the chain rule to obtain $\frac{dx}{dy}=e^t\frac{dt}{dy}$ A1: $\frac{dy}{dx}=e^{-t}\frac{dy}{dt}$ oe	M1A1
	$\frac{dy}{dx}=x^{-1}\frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2}=-x^{-2}\frac{dy}{dt}+x^{-1}\frac{d^2y}{dt^2}\cdot\frac{dt}{dx}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only A1: Correct second derivative oe	dM1A1
	$x^2\left(\frac{1}{x^2}\frac{d^2y}{dt^2}-\frac{1}{x^2}\frac{dy}{dt}\right)-3x\left(\frac{1}{x}\frac{dy}{dt}\right)+3y=\left(e^t\right)^2$	Substitutes their $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ in terms of t into the differential equation	M1
	$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+3y=e^{2t}$	cso	A1
			(6)

Alternative		
$x=e^t \Rightarrow \frac{dy}{dt}=e^t\frac{dy}{dx}=x\frac{dy}{dx}$	M1: Attempt first derivative using $\frac{dy}{dt}=\frac{dx}{dt}\times\frac{dy}{dx}$ A1: $\frac{dy}{dt}=x\frac{dy}{dx}$ oe	M1A1
$\frac{d^2y}{dt^2}=\frac{dx}{dt}\frac{dy}{dx}+x\frac{d^2y}{dx^2}\cdot\frac{dx}{dt}=x\frac{dy}{dx}+x^2\frac{d^2y}{dx^2}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only A1: Correct second derivative oe	dM1A1
$\frac{d^2y}{dt^2}-x\frac{dy}{dx}-3x\frac{dy}{dx}+3y=e^{2t}$ $=\frac{d^2y}{dt^2}-\frac{dy}{dt}-3\frac{dy}{dt}+3y=e^{2t}$	Substitutes their $\frac{d^2y}{dx^2}$ and $x\frac{dy}{dx}$ in terms of t into the differential equation	M1
$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+3y=e^{2t}$	Cso	A1
		(6)

(b)	$m^2 - 4m + 3 = 0 \Rightarrow m = 1,\ 3$	Solves (according to the General Guidance) the correct quadratic (so should be $m = \pm 1, \pm 3$)	M1
	$(y =)\ Ae^{3t} + Be^{t}$	Correct CF in terms of t not x. (May be seen later in their GS)	A1
	$y = ke^{2t}, y' = 2ke^{2t},\ y'' = 4ke^{2t}$	Correct form for PI and differentiates twice to obtain multiples of e^{2t} each time but do not allow if they are clearly integrating.	M1
	$4ke^{2t} - 8ke^{2t} + 3ke^{2t} = e^{2t} \Rightarrow k = \ldots$	Substitutes their $y,\ y',\ y''$ that are of the form αe^{2t} into the differential equation and sets = e^{2t} and proceeds to find their k	M1
	$(y) = -e^{2t}$	Correct PI or $k = -1$	A1
	$y = Ae^{3t} + Be^{t} - e^{2t}$	Correct ft GS **in terms of t** (their CF + their PI with non-zero PI). **Must be $y = \ldots$**	B1ft
			(6)
(c)	$(y =)\ Ax^3 + Bx - x^2$	Allow equivalent expressions in terms of x e.g. $(y =)\ Ae^{3\ln x} + Be^{\ln x} - e^{2\ln x}$. **Note that $y = \ldots$ is not needed here.**	B1
			(1)
			Total 13

Question Number	Scheme	Marks
7(a)	$z=y^{-2}\Rightarrow$ $\frac{dz}{dy}=-2y^{-3}$ or $1=-2y^{-3}\frac{dy}{dz}$ $y^2=z^{-1}\Rightarrow$ $2y\frac{dy}{dz}=-z^{-2}$ $y=z^{-\frac{1}{2}}\Rightarrow$ $\frac{dy}{dz}=-\frac{1}{2}z^{-\frac{3}{2}}$	B1
	$\frac{dy}{dx}=\frac{dz}{dx}\cdot\frac{dy}{dz}$ (oe) $\Rightarrow$ e.g., $\frac{dy}{dx}=-\frac{1}{2}y^3\frac{dz}{dx}$, $\frac{dy}{dx}=-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	M1 A1
	$x\frac{dy}{dx}+y+4x^2y^3\ln x=0\Rightarrow$ e.g., $-\frac{1}{2}xy^3\frac{dz}{dx}+y+4x^2y^3\ln x=0$ $-\frac{1}{2}xz^{-\frac{3}{2}}\frac{dz}{dx}+z^{-\frac{1}{2}}+4x^2z^{-\frac{3}{2}}\ln x=0$	dM1
	$\frac{dz}{dx}-\frac{2}{xy^2}-8x\ln x=0$ $\frac{dz}{dx}-\frac{2z}{x}-8x\ln x=0$ $\Rightarrow\frac{dz}{dx}-\frac{2z}{x}=8x\ln x$ *	A1*
		(5)
(b)	(IF =) $e^{\int-\frac{2}{x}(dx)}$	M1
	$=e^{-2\ln x}$ $\left(=x^{-2}\right)$	A1
	$x^{-2}z=\int x^{-2}(8x\ln x)[dx]$	M1
	E.g. Parts: $\int x^{-1}\ln x\,dx$: $\left(u=\ln x, u'=x^{-1}, v'=x^{-1}, v=\ln x\right)$ $\Rightarrow I=(\ln x)^2-kI\Rightarrow I=p(\ln x)^2$ Or substitution: $t=\ln x$, $\frac{dt}{dx}=x^{-1}$ $\Rightarrow I=k\int x^{-1}\ln x\cdot x\,dt=k\int t\,dt=pt^2$	M1
	$\int x^{-1}\ln x\,dx=\frac{1}{2}(\ln x)^2$ [+c]	A1
	$x^{-2}z=4(\ln x)^2+k\Rightarrow z=4x^2(\ln x)^2+kx^2$ $\Rightarrow y^2=\frac{1}{4x^2(\ln x)^2+kx^2}$ oe	A1
		(6)
		Total 11

Question Number	Scheme	Notes	Marks
4(a)		M1: A circle anywhere.	M1A1
		A1: A circle **correctly positioned** with centre –i or -1 marked in the correct place or (0, -1) or (-1, 0) or (0, -i) or (–i, 0) marked in the **correct place** and passing through (0, 0). The centre may be indicated away from the sketch but the sketch takes precedence. Ignore any shading.	
			(2)

(b) **Way 1**	$w=\frac{3\mathrm{i}z-2}{z+\mathrm{i}}$		
	$z=\frac{w\mathrm{i}+2}{3\mathrm{i}-w}$	M1: Attempt to make z the subject A1: Correct rearrangement oe	M1A1
	$z+\mathrm{i}=\frac{w\mathrm{i}+2}{3\mathrm{i}-w}+\mathrm{i}=\frac{w\mathrm{i}+2-3-w\mathrm{i}}{3\mathrm{i}-w}$	Applies $z+\mathrm{i}$ and finds common denominator	M1
	$\left\|\frac{w\mathrm{i}+2-3-w\mathrm{i}}{3\mathrm{i}-w}\right\|=1$	M1: Sets $\|z+\mathrm{i}\|=1$ A1: Correct equation, simplified or unsimplified	M1A1
	Note if they work with $w=u+\mathrm{i}v$ they should reach $\left\|\frac{2-v+u\mathrm{i}-u\mathrm{i}-(3-v)}{-u+(3-v)\mathrm{i}}\right\|=1$ *		
	$\left\|\frac{-1}{3\mathrm{i}-w}\right\|=1\Rightarrow\|w-3\mathrm{i}\|=1\Rightarrow\|u+\mathrm{i}v-3\mathrm{i}\|=1$ $\Rightarrow u^2+(3-v)^2=1$ or equivalent e.g. $u^2+(v-3)^2=1,\ u^2+v^2-6v+9=1$ **d**M1: Introduces u and v or x and y (may occur earlier *) and uses Pythagoras correctly to find a Cartesian form **<u>This mark is dependent on all the previous method marks</u>** A1: Correct equation (allow u, v or x, y or a, b)		**d**M1A1
			(7)

Question Number	Scheme	Notes	Marks
1	$2(\cos 0+\mathrm{i}\sin 0)$ or 2	$(z=)\ 2$ or $(z=)\ 2(\cos 0+\mathrm{i}\sin 0)$ or $2\cos 0+\mathrm{i}\sin 0$ or $2+0\mathrm{i}$ Allow $2(\cos 0\pi+\mathrm{i}\sin 0\pi)$	B1
	$2\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)$	This answer in this form. Do not allow e.g. $2e^{\frac{2\pi}{5}\mathrm{i}}$ but allow $2\cos\frac{2\pi}{5}+2\mathrm{i}\sin\frac{2\pi}{5}$	B1
	$2\left(\cos\frac{2k\pi}{5}+\mathrm{i}\sin\frac{2k\pi}{5}\right),(k=2,3,4)$	Attempts at least 2 more solutions whose arguments differ by $\frac{2\pi}{5}$. Allow this mark if the arguments are out of range. May be implied by their answers.	M1
	Note that this answer in general solution form can score full marks if correct i.e. the A marks below can be implied. E.g. $z=2\left(\cos\frac{2k\pi}{5}+\mathrm{i}\sin\frac{2k\pi}{5}\right),(k=0,1,2,3,4)$ scores full marks		

	$2\left(\cos\frac{4\pi}{5}+\text{i}\sin\frac{4\pi}{5}\right)$ $2\left(\cos\frac{6\pi}{5}+\text{i}\sin\frac{6\pi}{5}\right)$ $2\left(\cos\frac{8\pi}{5}+\text{i}\sin\frac{8\pi}{5}\right)$	A1: One further correct answer, allow the brackets to be expanded. A1: All correct, allow the brackets to be expanded.	A1 A1
Do not allow $2\left(\cos\frac{4\pi}{5}-\text{i}\sin\frac{4\pi}{5}\right)$ or $2\left(\cos\left(-\frac{4\pi}{5}\right)+\text{i}\sin\left(-\frac{4\pi}{5}\right)\right)$ for $2\left(\cos\frac{6\pi}{5}+\text{i}\sin\frac{6\pi}{5}\right)$ Do not allow $2\left(\cos\frac{2\pi}{5}-\text{i}\sin\frac{2\pi}{5}\right)$ or $2\left(\cos\left(-\frac{2\pi}{5}\right)+\text{i}\sin\left(-\frac{2\pi}{5}\right)\right)$ for $2\left(\cos\frac{8\pi}{5}+\text{i}\sin\frac{8\pi}{5}\right)$			
Ignore answers outside the range. For a fully correct solution that has extra solutions in range, deduct the final A mark.			
Answers in **degrees**: Penalise once the first time it occurs. Answers in degrees are: 0, 72, 144, 216, 288			
			(5)
			Total 5

Question Number	Scheme		Marks
8.	$x=\text{e}^t$		
(a)	$\frac{\text{d}y}{\text{d}x}=\frac{\text{d}y}{\text{d}t}\times\frac{\text{d}t}{\text{d}x}$	Attempt to use an appropriate version of the chain rule	M1
	$\frac{\text{d}y}{\text{d}x}=\frac{\text{d}y}{\text{d}t}\times\frac{1}{\text{e}^t}\left(=\frac{1}{x}\frac{\text{d}y}{\text{d}t}\right)$	Oe	A1
	$\frac{\text{d}^2y}{\text{d}x^2}=-\frac{1}{x^2}\frac{\text{d}y}{\text{d}t},+\frac{1}{x}\frac{\text{d}^2y}{\text{d}t^2}\frac{\text{d}t}{\text{d}x}$ or $\frac{\text{d}^2y}{\text{d}x^2}=-\frac{1}{x^2}\frac{\text{d}y}{\text{d}t},+\frac{1}{x^2}\frac{\text{d}^2y}{\text{d}t^2}$	M1: Use of the product rule (penalise chain rule errors by loss of A mark or marks) $\left(\text{Note}\quad t=\ln x\Rightarrow\frac{\text{d}t}{\text{d}x}=\frac{1}{x}\right)$	M1 A1, A1
	$x^2\frac{\text{d}^2y}{\text{d}x^2}+5x\frac{\text{d}y}{\text{d}x}+13y=0$ $\Rightarrow x^2.\frac{1}{x^2}\left(\frac{\text{d}^2y}{\text{d}t^2}-\frac{\text{d}y}{\text{d}t}\right)+5x\frac{1}{x}\frac{\text{d}y}{\text{d}t}+13y=0$ $\Rightarrow\frac{\text{d}^2y}{\text{d}t^2}+4\frac{\text{d}y}{\text{d}t}+13y=0$*	M1: Substitutes their first and second derivatives into the given differential equation Depends on both M marks above A1: Correct completion to printed answer	ddM1A1
			(7)
(b)	$m^2+4m+13=0$ $\Rightarrow(m=)\frac{-4\pm\sqrt{16-52}}{2}$	Attempt to solve the auxiliary equation	M1
	$(m=)-2\pm3\text{i}$	Correct roots May be implied by a correct GS	A1
	$y=\text{e}^{-2t}(A\cos3t+B\sin3t)$ or $y=A\text{e}^{(-2+3\text{i})t}+B\text{e}^{(-2-3\text{i})t}$	Correct GS	A1
	$t=\ln x$		B1
	$y=\frac{A\cos(3\ln x)+B\sin(3\ln x)}{x^2}$ or $y=A\text{e}^{(-2+3\text{i})\ln x}+B\text{e}^{(-2-3\text{i})\ln x}$		A1
			(5)
			Total 12

6	$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+13x=8e^{-3t} \qquad t\ldots0$		
(a)	$m^2+6m+13=0 \Rightarrow m=\frac{-6\pm\sqrt{36-52}}{2}$ $\{=-3\pm 2i\}$	Forms correct auxiliary equation and obtains a correct numerical expression for at least one root by formula or uses CTS (apply usual CTS rule below). One correct root if no working	**M1**
	CTS rule : $m^2+6m+13=0 \Rightarrow \left(m\pm\frac{6}{2}\right)^2 \pm q\pm 13=0,\ q\neq 0 \Rightarrow m=\ldots$		
	CF examples: $(x=)\ e^{-3t}\left(A\cos 2t+B\sin 2t\right)$ or $(x=)\ Ae^{-3t}\cos(-2t)+Be^{-3t}\sin(-2t)$ or $(x=)\ Pe^{(-3+2i)t}+Qe^{(-3-2i)t}$ or $(x=)\ e^{-3t}\left(Pe^{2it}+Qe^{-2it}\right)$	Correct complementary function in any form, allow if the "$x=$" is missing or wrong and accept for this mark if the CF is given fully in terms of x instead of t.	**A1**
	PI: $\{x=\}\ \lambda e^{-3t}$	Correct form for the particular integral selected. Must include λe^{-3t} but accept with any extra terms that correctly disappear when coefficients found. Accept "PI=". If λe^{pt} is used $p=-3$ must be seen later.	**B1**
	$\frac{dx}{dt}=-3\lambda e^{-3t};\ \frac{d^2x}{dt^2}=9\lambda e^{-3t}$ $\Rightarrow 9\lambda e^{-3t}+6\left(-3\lambda e^{-3t}\right)+13\lambda e^{-3t}=8e^{-3t}$	Differentiates a PI of **any** form twice (**provided** it has at least one constant and is a function of t) **and** substitutes into the equation. Allow only sign/coefficient errors only in the differentiation. Their PI must lead to non-zero derivatives.	**M1**
	$\Rightarrow 9\lambda-18\lambda+13\lambda=8 \Rightarrow \lambda=\ldots\ (2)$	Proceeds to find the value of the constant **following use of a PI of the correct form**. Any unnecessary extra terms in the PI must be found to be zero	**dM1**
	$x="e^{-3t}\left(A\cos 2t+B\sin 2t\right)"+2e^{-3t}$	Correct general solution ft on **their CF only** – any CF **provided** it has at least one constant and is in terms of t. **Must have $x=\ldots$** Do not allow if their CF is miscopied or mathematically changed	**A1ft**
	Work with a PI of the form λte^{-3t} is B0M1dM0A0 max even if $2e^{-3t}$ is obtained. Only condone incorrect variables if they are recovered but refer to the note for the first A1.		**(6)**

6(b)	$x=\frac{1}{2}$ at $t=0$ $\Rightarrow \frac{1}{2}=A+2 \quad \left(\Rightarrow A=-\frac{3}{2}\right)$	Uses the initial condition for x in their GS to find a linear equation in one or two constants. Allow for GS = CF or CF + PI and the constant may come from the +PI	M1
	$x=\mathrm{e}^{-3t}\left(A\cos 2t+B\sin 2t\right)+2\mathrm{e}^{-3t}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{e}^{-3t}\left(-2A\sin 2t+2B\cos 2t\right)-3\mathrm{e}^{-3t}\left(A\cos 2t+B\sin 2t\right)-6\mathrm{e}^{-3t}$ Uses the **product rule** to differentiate their real GS obtaining an expression in terms of t of the correct form for their GS (sign and coefficient errors only – so do not allow e.g., $...\mathrm{e}^{pt}\to...\mathrm{e}^{qt}$). Allow for GS = CF or CF + PI and does not have to include constants. If they work with a complex function e.g., $x=P\mathrm{e}^{(-3+2\mathrm{i})t}+Q\mathrm{e}^{(-3-2\mathrm{i})t}+2\mathrm{e}^{-3t}$ progress is unlikely. This mark is not scored for work in (c)		M1
	$t=0, \frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2}\Rightarrow\frac{1}{2}=2B-3A-6\Rightarrow B=...(=1)$ Uses both initial conditions to find values for the 2 constants (no others) in their **GS = (CF with 2 constants) + PI(no constants)**. One constant must be found to be non-zero. **Requires both previous M marks.**		ddM1
	Examples: $x=\mathrm{e}^{-3t}\left(-\frac{3}{2}\cos 2t+\sin 2t\right)+2\mathrm{e}^{-3t}$ or $x=\mathrm{e}^{-3t}\left(-\frac{3}{2}\cos 2t+\sin 2t+2\right)$ or $x=2\mathrm{e}^{-3t}-\frac{3}{2}\mathrm{e}^{-3t}\cos 2t+\mathrm{e}^{-3t}\sin 2t$	Correct particular solution in any form in terms of t. **Must be x = ...** unless this was the only reason for final A0 in part (a) due to omission or e.g, "y = ..." was used	A1
			(4)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{e}^{-3t}\left(3\sin 2t+2\cos 2t\right)-3\mathrm{e}^{-3t}\left(-\frac{3}{2}\cos 2t+\sin 2t\right)-6\mathrm{e}^{-3t}=0$ Sets an expression for $\frac{\mathrm{d}x}{\mathrm{d}t}=0$. Accept with any unfound constants provided $\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{f}(t)$		M1
	$\left(3\sin 2t+2\cos 2t\right)-3\left(-\frac{3}{2}\cos 2t+\sin 2t\right)-6=0$ Achieves an equation of the form $a\sin bt+c\cos bt+d=0$ <u>or equivalent with terms uncollected</u>. One of a and c non-zero and b and d non-zero. **Must follow a GS = CF + PI where two constants were found for the CF and one for the PI. Requires previous M mark.**		dM1
	$\cos 2t=\frac{12}{13}\Rightarrow t=0.1973955598...\Rightarrow x$ or $a=\frac{1}{2}\mathrm{e}^{-3(0.1973...)}\left(4-3\times\frac{12}{13}+2\sin(2\times 0.1973...)\right)=..$ Finds a value of t from cos kt = c $\left(k\neq 1, -1<c<1\right)$ and uses their positive (or made positive) value of t to find a value of x (or a) via their PS. Accept a pair of stated values. **Requires both previous M marks.**		ddM1
	x or $a=0.553(1164729...)$	awrt 0.553	A1
			(4)
			Total 14

1(a)	$r=\sqrt{(-4)^2+\left(-4\sqrt{3}\right)^2}=\ldots$	M1
	$\tan\theta=\dfrac{-4\sqrt{3}}{-4}\Rightarrow\theta=\tan^{-1}\left(\sqrt{3}\right)\pm\pi$	M1
	$8\left(\cos\left(-\dfrac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\dfrac{2\pi}{3}\right)\right)$	A1
		(3)
(b)	$z=r\mathrm{e}^{\mathrm{i}\theta}\Rightarrow\left(r\mathrm{e}^{\mathrm{i}\theta}\right)^3=-4-4\sqrt{3}\Rightarrow r^3\left(\mathrm{e}^{3\mathrm{i}\theta}\right)=8\mathrm{e}^{-\mathrm{i}\frac{2\pi}{3}}$	
	$\Rightarrow r=\sqrt[3]{8}=2$	M1
	$3\theta=-\dfrac{2\pi}{3}(+2k\pi)\Rightarrow\theta=-\dfrac{2\pi}{9}+\left(\dfrac{2k\pi}{3}\right)$	M1
	So $z=2\mathrm{e}^{-\frac{8\pi}{9}\mathrm{i}},2\mathrm{e}^{-\frac{2\pi}{9}\mathrm{i}},2\mathrm{e}^{\frac{4\pi}{9}\mathrm{i}}$	A1ft A1
		(4)
		(7 marks)