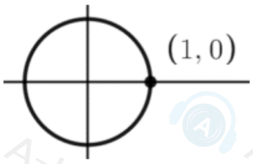
Question Number	Scheme	Notes	Marks
3	$\frac{dy}{dx}+2xy=xe^{-x^2}y^3$		
(a)	$z=y^{-2}\Rightarrow y=z^{-\frac{1}{2}}$		
	$\frac{dy}{dx}=-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	M1: $\frac{dy}{dx}=kz^{-\frac{3}{2}}\frac{dz}{dx}$ A1: Correct differentiation	M1A1
	$-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}+\frac{2x}{z^{\frac{1}{2}}}=xe^{-x^2}z^{-\frac{3}{2}}$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1cso
			(4)
	(a) Alternative 1		
	$\frac{dz}{dy}=-2y^{-3}$ oe	M1: $\frac{dz}{dy}=ky^{-3}$ A1: Correct differentiation	M1A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
	(a) Alternative 2		
	$\frac{dz}{dx}=-2y^{-3}\frac{dy}{dx}$	M1: $\frac{dz}{dx}=ky^{-3}\frac{dy}{dx}$ inc chain rule A1: Correct differentiation	M1A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
(b)	$I=e^{\int -4x\,dx}=e^{-2x^2}$	M1: $I=e^{\int \pm 4x\,dx}$ A1: e^{-2x^2}	M1A1
	$ze^{-2x^2}=\int -2xe^{-3x^2}\,dx$	$z\times I=\int -2xe^{-x^2}I\,dx$	dM1
	$\frac{1}{3}e^{-3x^2}(+c)$	$\int xe^{qx^2}\,dx=pe^{qx^2}(+c)$	M1
	$z=ce^{2x^2}+\frac{1}{3}e^{-x^2}$	Or equivalent	A1
			(5)
(c)	$\frac{1}{y^2}=ce^{2x^2}+\frac{1}{3}e^{-x^2}\Rightarrow y^2=\frac{1}{ce^{2x^2}+\frac{1}{3}e^{-x^2}}$	$y^2=\frac{1}{(b)}\left(=\frac{3e^{x^2}}{1+ke^{3x^2}}\right)$	B1ft
			(1)
			Total 10

Question Number	Scheme	Notes	Marks
2(a)	$\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}+xy-x=0$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{xy}{\left(1+x^2\right)}=\frac{x}{\left(1+x^2\right)}$	Correct form.	B1
	$I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{\frac{1}{2}\ln\left(1+x^2\right)}=\left(1+x^2\right)^{\frac{1}{2}}$	M1: $I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{k\ln\left(1+x^2\right)}$ where k is a constant. (Condone missing brackets around the x^2+1) A1: Correct integrating factor of $\left(1+x^2\right)^{\frac{1}{2}}$	M1A1
	$y\left(1+x^2\right)^{\frac{1}{2}}=\int\frac{x}{\left(1+x^2\right)^{\frac{1}{2}}}\mathrm{d}x$	Uses their integration factor to reach the form $yI=\int Q\,\mathrm{I}\,\mathrm{d}x$	M1
	$=\left(1+x^2\right)^{\frac{1}{2}}(+c)$	Correct integration (+ c not needed)	A1
	$y=1+c\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao with the constant correctly placed. (The "$y=$" must appear at some point)	A1
			(6)

(b)	$2=1+c\left(1+3^2\right)^{-\frac{1}{2}}\Rightarrow c=\ldots$	Substitutes $x=3$ and $y=2$ and attempts to find a value for c.	M1
	$(y=)1+\sqrt{10}\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao. ("$y=$" not needed for this mark) and apply isw if necessary.	A1
			(2)
			Total 8

7 (a)	$x = r\cos\theta = 3\sin 2\theta\cos\theta$	B1	
	$\frac{dx}{d\theta} = 6\cos 2\theta\cos\theta - 3\sin 2\theta\sin\theta = 0$	M1	
	$2\cos\theta\left(\cos^2\theta - 2\sin^2\theta\right) = 0$	M1	
ALT	For the 2 M marks:		
	$x = 6\sin\theta\cos^2\theta \Rightarrow \frac{dx}{d\theta} = 6\cos^3\theta - 12\sin^2\theta\cos\theta = 0$		
	$\tan\phi = \frac{1}{\sqrt{2}}$ *	A1*	(4)
(b)	$\tan\phi = \frac{1}{\sqrt{2}} \Rightarrow \sin\phi = \frac{1}{\sqrt{3}},\ \cos\phi = \frac{\sqrt{2}}{\sqrt{3}}$	M1	
	$R = 3\times 2\times\frac{1}{\sqrt{3}}\times\frac{\sqrt{2}}{\sqrt{3}} = 2\sqrt{2}$	A1	(2)
(c)	Area of sector $= \frac{1}{2}\int r^2 d\theta = \frac{9}{2}\int \sin^2 2\theta\, d\theta$	M1	
	$= \frac{9}{2}\int_0^{\arctan\left(\frac{1}{\sqrt{2}}\right)} \frac{1}{2}(1-\cos 4\theta)\,d\theta$	M1	
	$= \frac{9}{2}\left[\frac{1}{2}\left(\theta - \frac{1}{4}\sin 4\theta\right)\right]_0^{\arctan\frac{1}{\sqrt{2}}}$	M1A1	
	$= \frac{9}{4}\left[\arctan\frac{1}{\sqrt{2}} - \frac{1}{4}\sin 4\left(\arctan\frac{1}{\sqrt{2}}\right) - 0\right]$	dM1	
	$\sin 4\phi = 2\sin 2\phi\cos 2\phi = 4\sin\phi\cos\phi\left(2\cos^2\phi - 1\right)$ $= 4\times\frac{1}{\sqrt{3}}\times\frac{\sqrt{2}}{\sqrt{3}}\left(2\times\frac{2}{3}-1\right) = \frac{4\sqrt{2}}{9}$	M1	
	Area of sector $= \frac{9}{4}\left(\arctan\frac{1}{\sqrt{2}} - \frac{1}{4}\times\frac{4\sqrt{2}}{9}\right) = \frac{9}{4}\arctan\frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{4}$	A1	(7)
			[13]

Question Number	Scheme	Notes	Marks
3(a)		Circle centred at origin with radius of 1 correctly indicated in any way. May use $\pm\text{i}$ or ± 1 in appropriate position on imaginary axis.	B1
			(1)
(b)	$w = \frac{9\text{i}z - \text{i}}{z+1}$ $\Rightarrow wz + w = 9\text{i}z - \text{i} \Rightarrow z(w - 9\text{i}) = -w - \text{i}$ $\Rightarrow z = \frac{-w-\text{i}}{w-9\text{i}} = \frac{w+\text{i}}{9\text{i}-w}$ oe	M1: Makes z the subject A1: Correct expression oe	M1 A1
	$\lvert z\rvert = \frac{\lvert -w-\text{i}\rvert}{\lvert w-9\text{i}\rvert} \Rightarrow \lvert w-9\text{i}\rvert = \lvert w-(-\text{i})\rvert$	Uses $\lvert z\rvert = 1$ and obtains $\lvert \pm w \pm z_1\rvert = \lvert \pm w \pm z_2\rvert$ **Requires previous M mark.**	**d**M1
	$\Rightarrow v = "4"$ or $u^2 + (v-9)^2 = u^2 + (v+1)^2$ $-18v + 81 = 2v + 1$ $20v = 80$ $v = 4$	Obtains $v = k$ with k correct for their $\lvert w - p\text{i}\rvert = \lvert w - q\text{i}\rvert$ oe or substitutes $u + \text{i}v$ for w, applies Pythagoras correctly and obtains a value for v Allow "$y = ...$" for this mark. Condone $v = 4\text{i}$ for this mark. **Requires both previous M marks.**	**dd**M1
	$v = 4$ only	Correct equation	A1
			(5)

Question Number	Scheme	Marks
5(a)	$w = \frac{z+1}{z-3} \Rightarrow wz - 3w = z + 1 \Rightarrow wz - z = 3w + 1 \Rightarrow z = \frac{3w+1}{w-1}$	M1 A1
	$(z =) \frac{(3u+1) + 3\text{i}v}{(u-1) + \text{i}v} \times \frac{(u-1) - \text{i}v}{(u-1) - \text{i}v} = ...$	**d**M1
	$\Rightarrow x = \frac{3u^2 - 2u + 3v^2 - 1}{(u-1)^2 + v^2} \quad y = \frac{-4v}{(u-1)^2 + v^2}$ $(y = 4x \Rightarrow) \quad -4v = 12u^2 - 8u + 12v^2 - 4$	**dd**M1
	$\Rightarrow 3u^2 + 3v^2 - 2u + v - 1 = 0$*	A1cso*
		(5)
(b)	$u^2 - \frac{2}{3}u + v^2 + \frac{1}{3}v - \frac{1}{3} = 0 \Rightarrow \left(u - \frac{1}{3}\right)^2 - \frac{1}{9} + \left(v + \frac{1}{6}\right)^2 - \frac{1}{36} - \frac{1}{3} = 0$ $\Rightarrow \left(u - \frac{1}{3}\right)^2 + \left(v + \frac{1}{6}\right)^2 = \frac{17}{36}$ $\Rightarrow$ centre: $\left(\frac{1}{3}, -\frac{1}{6}\right)$ $\left[\text{allow } x = \frac{1}{3}, y = -\frac{1}{6}\right]$ radius: $\sqrt{\frac{17}{36}}$ or $\frac{\sqrt{17}}{6}$	B1 B1
		(2)
		Total 7

1.	$\mathrm{i}\left(1+\sqrt{3}\right)=\dfrac{\mathrm{i}\left(1+\sqrt{3}\right)+p\mathrm{i}}{\mathrm{i}^2\left(1+\sqrt{3}\right)+3}$	M1
	$-\mathrm{i}\left(1+\sqrt{3}\right)^2+3\mathrm{i}\left(1+\sqrt{3}\right)=\mathrm{i}\left(1+\sqrt{3}\right)+p\mathrm{i}$	
	$-1-2\sqrt{3}-3+3+3\sqrt{3}=1+\sqrt{3}+p$	dM1
	$p=-2$	A1
		[3]

3(a)	$\dfrac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}}\times\dfrac{\sqrt{r(r+1)}-\sqrt{r(r-1)}}{\sqrt{r(r+1)}-\sqrt{r(r-1)}}$	A correct multiplier to rationalise the denominator seen or implied by correct work	**M1**
	$=\dfrac{r\left(\sqrt{r(r+1)}-\sqrt{r(r-1)}\right)}{r(r+1)-r(r-1)}=\dfrac{\sqrt{r(r+1)}-\sqrt{r(r-1)}}{2}$ or $A=\dfrac{1}{2}$ Correct expression or correct value for A. Condone poor notation if intention clear. There must be (minimal) correct supporting working.		**A1**
	Alternative: $A=\dfrac{r}{\left(\sqrt{r(r+1)}+\sqrt{r(r-1)}\right)\left(\sqrt{r(r+1)}-\sqrt{r(r-1)}\right)}=\dfrac{r}{r(r+1)-r(r-1)}$ or $\dfrac{r}{r^2+r-r^2+r}$ or $\dfrac{r}{2r}\Rightarrow A=\dfrac{1}{2}$ M1: Correctly makes A the subject A1: Correct completion with one intermediate fraction		
			(2)
(b)	$\sum_{r=1}^{n}\dfrac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}}=\text{"}\dfrac{1}{2}\text{"}\left(\begin{array}{c}\sqrt{1\times2}-\sqrt{1\times0}\ \left(=\sqrt{2}\ (-0)\right)\\ +\sqrt{2\times3}-\sqrt{2\times1}\ \left(=\sqrt{6}-\sqrt{2}\right)+\ldots\\ \ldots+\sqrt{(n-1)(n-1+1)}-\sqrt{(n-1)(n-1-1)}\ \left(=\sqrt{n(n-1)}-\sqrt{(n-1)(n-2)}\right)\\ +\sqrt{n(n+1)}-\sqrt{n(n-1)}\end{array}\right)$ **M1**: Applies the method of differences for $r=1$ and $r=n$ in the given expression with or without their A and obtains one correct row of these 2. **M1**: Applies the method of differences for $r=1$, $r=n$ and **either** $r=2$ or $r=n-1$ in the given expression with/without their A and obtains 2 correct rows of these 4.		

	When considering how many rows are correct, if A has been clearly applied to any term then assess all rows as if A has been applied throughout. Condone missing bracket if their A is applied to a row e.g., "$\frac{1}{2}\times\ \sqrt{6}-\sqrt{2}$" if it is recovered but e.g., $\frac{\sqrt{6}}{2}-\sqrt{2}$ is an incorrect row. Ignore a row for $r=0$. Condone equivalent work with r or e.g., k used for n. **Both marks can be implied by a correct final expression with or without their A provided there are at least any two correct rows of differences** i.e., $"\frac{1}{2}"\left(\sqrt{n(n+1)}-0\right)$ or $\sqrt{n(n+1)}-0$		M1 M1
	Note: row 3 is $"\frac{1}{2}"(\sqrt{12}\,(\text{or }2\sqrt{3})-\sqrt{6})$, row 4 is $"\frac{1}{2}"(\sqrt{20}\,(\text{or }2\sqrt{5})-\sqrt{12}\,(\text{or }2\sqrt{3}))$ If $\frac{1}{2}$ is fully applied the rows are: $\frac{\sqrt{2}}{2},\ \frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2},\ \frac{\sqrt{12}}{2}(\text{or }\sqrt{3})-\frac{\sqrt{6}}{2},\ \frac{\sqrt{20}}{2}(\text{or }\sqrt{5})-\frac{\sqrt{12}}{2}(\text{or }\sqrt{3}),\ldots$ $\ldots,\frac{\sqrt{(n-2)(n-1)}}{2}-\frac{\sqrt{(n-2)(n-3)}}{2},\ \frac{\sqrt{n(n-1)}}{2}-\frac{\sqrt{(n-1)(n-2)}}{2},\ \frac{\sqrt{n(n+1)}}{2}-\frac{\sqrt{n(n-1)}}{2}$		
	$=\frac{1}{2}\sqrt{n(n+1)}$ oe e.g., $\frac{\sqrt{n^2+n}}{2}$	Correct expression in terms of n. No incorrect terms seen in differences work even if cancelled but condone the occasional poor bracket. There should be no "0" so e.g., $\frac{1}{2}\left(\sqrt{n(n+1)}-0\right)$ is A0 Does not require marks in (a)	A1
			(3)

Question Number	Scheme	Notes	Marks
3(c)	$\sum r=\frac{1}{2}n(n+1)$ e.g., sight of $k\times\ldots=\sqrt{\frac{1}{2}n(n+1)}$	States or uses the correct summation formula for integers	**M1**
	$\frac{k}{2}\sqrt{n(n+1)}=\sqrt{\frac{1}{2}n(n+1)}\Rightarrow\frac{k}{2}=\sqrt{\frac{1}{2}}\Rightarrow k=\sqrt{2}$	$\sqrt{2}$ only (Not $\pm$). $k=\sqrt{2}$ must not come from a clearly incorrect equation.	**A1**
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
1	$\frac{1}{x-2}>\frac{2}{x}$		
	$\frac{1}{x-2}-\frac{2}{x}>0\Rightarrow\frac{4-x}{x(x-2)}>0$	Collect to one side and attempt common denominator of $x(x-2)$	M1
	$x=\underline{0,\ 2,}\ \underline{\underline{4}}$	B1 for 0 **and** 2, A1 for 4	B1, A1
	$x<0,\quad 2<x<4$ For their critical values α,β and γ in ascending order, attempts $x<\alpha$ and $\beta<x<\gamma$ condoning the use of a mixture of open or closed inequalities or For **one of** $x<0$ **or** $2<x<4$ condoning the use of a mixture of open or closed inequalities		M1
	$x<0,\quad 2<x<4$ $(-\infty,0)$ or $[-\infty,0),\ (2,\ 4)$	Correct inequalities. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but not $\cap$	A1
			(5)
			Total 5

7(a)(i)+(ii)	$\cos 5\theta + i \sin 5\theta = (\cos\theta + i\sin\theta)^5$ o.e.	M1
	$= c^5 + 5c^4si - 10c^3s^2 - 10c^2s^3i + 5cs^4 + s^5i$ $\Rightarrow \sin 5\theta = ..., \cos 5\theta = ...$	M1
	$\sin 5\theta \equiv 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ *	A1*
	$\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	A1
		(4)
(b)	$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta} \times \frac{\cos^{-5}\theta}{\cos^{-5}\theta}$	M1
	$\frac{\frac{5\sin\theta}{\cos\theta} - \frac{10\sin^3\theta}{\cos^3\theta} + \frac{\sin^5\theta}{\cos^5\theta}}{1 - \frac{10\sin^2\theta}{\cos^2\theta} + \frac{5\sin^4\theta}{\cos^4\theta}} = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$	A1*
		(2)
(c)	$x = \tan\theta,\ 2x^5 - 15x^4 - 20x^3 + 30x^2 + 10x - 3 = 0$ $\Rightarrow \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta} = \frac{3}{2} \Rightarrow \tan 5\theta = \frac{3}{2}$	M1
	$\theta = \frac{1}{5}\tan^{-1}\frac{3}{2}$ $(= 0.19655 \text{ radians}, 11.2619...°)$ Others that you may see which would also score: In radians {0.19655 ..., 0.82487..., 1.45319..., 2.0815..., 2.7098...} In degrees {11.2619 ..., 47.2619 ..., 83.2619 ..., 119.2619 ..., 155.2619 ...}	A1
	$\Rightarrow x = \tan 0.196555..., ...$	M1
	Two of: $x = 0.1991..., 1.0822..., 8.464..., -1.7847..., -0.4607...$	A1
	All of: $x = -1.785, -0.461, 0.199, 1.082, 8.464$	A1
		(5)
		(11 marks)

2(a)	$x\frac{dy}{dx} - y^3 = 4 \Rightarrow \frac{dy}{dx} + x\frac{d^2y}{dx^2} - 3y^2\frac{dy}{dx} = 0$ Alt: $\frac{dy}{dx} = \frac{y^3}{x} + \frac{4}{x} \Rightarrow \frac{d^2y}{dx^2} = -\frac{y^3}{x^2} + \frac{3y^2}{x}\frac{dy}{dx} - \frac{4}{x^2}$	M1A1
	$\Rightarrow \frac{d^2y}{dx^2} + x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 6y\left(\frac{dy}{dx}\right)^2 - 3y^2\frac{d^2y}{dx^2} = 0$ Alt: $\Rightarrow \frac{d^3y}{dx^3} = \frac{2y^3}{x^3} - \frac{3y^2}{x^2}\frac{dy}{dx} - \frac{3y^2}{x^2}\frac{dy}{dx} + \frac{1}{x}\left(6y\left(\frac{dy}{dx}\right)^2 + 3y^2\frac{d^2y}{dx^2}\right) + \frac{8}{x^3}$	dM1
	$\Rightarrow x\frac{d^3y}{dx^3} = 6y\left(\frac{dy}{dx}\right)^2 + (3y^2 - 2)\frac{d^2y}{dx^2}$	A1
		(4)

(b)	$x=2,\ y=1 \Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=2} = \frac{4+1}{2} = \frac{5}{2},\ \left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_{x=2} = \frac{3(1)\left(\frac{5}{2}\right)-\frac{5}{2}}{2} = \frac{5}{2}$ $\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_{x=2} = \frac{6(1)\left(\frac{25}{4}\right)+\frac{5}{2}}{2} = 20$	M1
	$(y=)\mathrm{f}(2)+(x-2)\mathrm{f}'(2)+\frac{(x-2)^2}{2}\mathrm{f}''(2)+\frac{(x-2)^3}{6}\mathrm{f}'''(2)+\ldots$ $=1+\frac{5}{2}(x-2)+\frac{5}{2}\frac{(x-2)^2}{2}+20\frac{(x-2)^3}{6}+\ldots$	M1
	$(y=)1+\frac{5}{2}(x-2)+\frac{5}{4}(x-2)^2+\frac{10}{3}(x-2)^3+\ldots$	A1
		(3)
		Total 7

3	$w=\frac{z}{z+4\mathrm{i}}$	
	$w(z+4\mathrm{i})=z \Rightarrow z(1-w)=4\mathrm{i}w$ or $z=\frac{4\mathrm{i}w}{1-w}$ oe	M1A1
	$\|z\|=3 \quad \left\|\frac{4\mathrm{i}w}{1-w}\right\|=3$	dM1
	$\|4\mathrm{i}w\|=3\|1-w\|$	
	$w=u+\mathrm{i}v \quad 16\left(u^2+v^2\right)=9\left((1-u)^2+v^2\right)$	ddM1A1
	$16u^2+16v^2=9\left(1-2u+u^2+v^2\right)$	
	$7u^2+7v^2+18u-9=0$	
	$\left(u+\frac{9}{7}\right)^2+v^2=\frac{144}{49}$	dddM1
	Centre $\left(-\frac{9}{7},0\right)$ Radius $\frac{12}{7}$	A1A1 (8)